Problems & Solutions

In honor of this year’s School Competition being released to registered competition coaches for our 2019-2020 Competition Series, see some of last year’s School Competition questions below!

Sprint Round, 19
Josie selected a number \( n \). She divided \( n \) by 2 and then subtracted \( \frac{1}{2} \) from the result. She took half of that result and then subtracted \( \frac{1}{2} \) to get the final result of 10. What is the value of \( n \)?

Let’s start by writing Josie’s first step as an expression. Dividing \( n \) by 2, then subtracting \( \frac{1}{2} \) from the result would be \( \frac{n}{2} - \frac{1}{2} \). If we build on this expression with the information in Josie’s second step, we get \( \frac{1}{2}(\frac{n}{2} - \frac{1}{2}) - \frac{1}{2} \). Knowing the final result is 10, we can set up the equation below, and solve for \( n \) as follows:

\[
\frac{1}{2}(\frac{n}{2} - \frac{1}{2}) - \frac{1}{2} = 10
\]

\[
\frac{1}{2}(\frac{n}{2} - \frac{1}{2}) - \frac{1}{2} = 10 \text{ (using the distributive property)}
\]

\[
\frac{n}{4} - \frac{1}{4} = 10 \text{ (multiplying to eliminate the parentheses)}
\]

\[
\frac{n}{4} - \frac{1}{4} = 10 \text{ (combining like terms)}
\]

\[
\frac{n}{4} = 10\frac{3}{4} \text{ (adding } \frac{3}{4} \text{ to both sides of the equation)}
\]

\[n = 43 \text{ (dividing both sides of the equation by } \frac{1}{4})\]

Target Round, 7
A box contains 4 chocolates and 1 fruit chew. Clark and Chloe take turns drawing a treat out of the box without replacement. Whoever draws the fruit chew wins. Clark draws first. What is the probability that Chloe wins? Express your answer as a common fraction.

If there are 5 total pieces of candy, this means there are 5 total turns to draw a piece out of the box. Since we are told Clark draws first and Chloe and Clark alternate draws, Chloe will only have 2 turns to draw. So, the probability she will draw the fruit chew is \( \frac{2}{5} \), because she would have two chances out of the total 5 turns to pick the fruit chew.

You can also look at the following possibilities to answer this question. If \( C = \) chocolate and \( F = \) fruit chew, the possible drawing scenarios are \( FCCCC \); \( CFCCC \); \( CCFCC \); \( CCCFC \); \( CCCCF \). Since we know that Clark draws first and Chloe and Clark alternate draws, Chloe’s choices would always be in the second and fourth positions in these scenarios. There are only 2 of the 5 scenarios in which the fruit chew is chosen on the second or fourth draw. Therefore, the probability that Chloe wins is \( \frac{2}{5} \).
Marcia wants to estimate the total weight of the sand in her sandbox. She has a drinking glass whose interior is a cylinder with diameter 2 inches and height 4 inches; the glass weighs 4.1 ounces when empty. After she fills the glass with sand, the weight of the glass is 14.8 ounces. If the sandbox is a rectangle measuring 6 feet by 4 feet, and the average depth of the sand is 6 inches, what is the total weight of the sand in the sandbox in pounds? Express your answer as a whole number to the nearest hundred. (1 foot = 12 inches; 1 pound = 16 ounces.)

Let’s start by finding the volume of the cylinder drinking glass. The formula for the volume of a cylinder is \( V = \pi r^2 h \). If the diameter of the cylinder is 2, then the radius is half that, so \( r = 1 \). We can then solve for the volume of the cylinder:

\[
V = \pi (1)^2 (4) \\
V = 4\pi \text{ in.}^3
\]

Now, let’s find the volume of the sand in the sandbox. The formula for the volume of a rectangular prism (like the sandbox) is \( V = l \times w \times h \) (where \( l = \) length, \( w = \) width, and \( h = \) height). We are provided all of these measurements in the problem, but be careful! We need to make sure they are all in the same units. Here, let’s put them all in inches. With 12 inches per 1 foot, 6 feet = 6 \times 12 = 72 inches, and 4 feet = 4 \times 12 = 48 inches. So, the volume of the sandbox is:

\[
V = 72 \times 48 \times 6 \\
V = 20,736 \text{ in.}^3
\]

In order to measure the weight of the sand, we’ll need to find how many glasses full of sand there will be, by dividing the volume of the sandbox by the volume of the glass. So, \( \frac{20,736 \text{ in.}^3}{4\pi \text{ in.}^3} \) is approximately 1,650.1184 glasses full of sand. Since the glass itself weighs 4.1 ounces, the number of ounces of sand in the glass when the glass is full is 14.8 – 4.1 = 10.7 ounces.

\[
1,650.1184 \text{ glasses} \times 10.7 \text{ ounces in each glass} = 17,656.2669 \text{ ounces of sand}
\]

Finally, knowing there are 16 ounces in 1 pound, \( 17,656.2669 \div 16 = 1,103.5167 \) pounds. This value rounded to the nearest hundred is 1,100 pounds.
Countdown Round, 21
Xanthia can read 100 pages per hour and Molly can read 50 pages per hour. If they each read the same book, and the book has 225 pages, how many more minutes than Xanthia would it take for Molly to finish reading the book?

At the rate of 100 pages per hour, Xanthia will read the 225-page book in \( \frac{225}{100} = 2.25 \) hours. With 60 minutes in 1 hour, this amount of time in minutes is \( 2.25 \times 60 = 135 \) minutes. At the rate of 50 pages per hour, Molly will read the same book in \( \frac{225}{50} = 4.5 \) hours. This amount of time in minutes is \( 4.5 \times 60 = 270 \) minutes.

\[ 270 - 135 = \boxed{135 \text{ minutes}} \text{ more that Molly takes to finish the book than Xanthia} \]

Countdown Round, 41
The operation ◊ is defined by \( a \diamond b = ab^2 - b + 1 \). What is the value of \((-1) \diamond 6\)?

\[
\begin{align*}
(-1) \diamond 6 &= (-1)(6)^2 - 6 + 1 \\
(-1) \diamond 6 &= (-1)(36) - 6 + 1 \\
(-1) \diamond 6 &= -36 - 6 + 1 \\
(-1) \diamond 6 &= -42 + 1 \\
(-1) \diamond 6 &= \boxed{-41}
\end{align*}
\]
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Sprint Round, 19
Josie selected a number $n$. She divided $n$ by 2 and then subtracted ½ from the result. She took half of that result and then subtracted ½ to get the final result of 10. What is the value of $n$?

Target Round, 7
A box contains 4 chocolates and 1 fruit chew. Clark and Chloe take turns drawing a treat out of the box without replacement. Whoever draws the fruit chew wins. Clark draws first. What is the probability that Chloe wins? Express your answer as a common fraction.

Team Round, 5
Marcia wants to estimate the total weight of the sand in her sandbox. She has a drinking glass whose interior is a cylinder with diameter 2 inches and height 4 inches; the glass weighs 4.1 ounces when empty. After she fills the glass with sand, the weight of the glass is 14.8 ounces. If the sandbox is a rectangle measuring 6 feet by 4 feet, and the average depth of the sand is 6 inches, what is the total weight of the sand in the sandbox in pounds? Express your answer as a whole number to the nearest hundred. (1 foot = 12 inches; 1 pound = 16 ounces.)

Countdown Round, 21
Xanthia can read 100 pages per hour and Molly can read 50 pages per hour. If they each read the same book, and the book has 225 pages, how many more minutes than Xanthia would it take for Molly to finish reading the book?

Countdown Round, 41
The operation ◊ is defined by $a ◊ b = ab^2 - b + 1$. What is the value of $(−1) ◊ 6$?