Problems

As November (the 11th month) gets underway, it’s the perfect time to focus on 11. Eleven is the fourth prime number, and there is a fun divisibility rule for 11. For any integer, insert alternating “–” and “+” signs between the consecutive pairs of digits, starting with a “–” sign between the left-most pair of digits. For example, for the number 91,828 we would have 9 – 1 + 8 – 2 + 8. (Notice that the first minus went between the left-most pair of numbers, 9 and 1, and then we alternated with “+” and “–” signs.) Now, simplify the expression. For our example, we have 9 – 1 + 8 – 2 + 8 = 22. Since this value, 22, is divisible by 11, the original number is divisible by 11. Using this rule, if the five-digit integer 76,9a2 is a multiple of 11, what is the value of a?

When playing many games, players must roll a pair of dice and find the sum of the two numbers rolled. With two dice, there are 11 possible sums ranging from 2 through 12. What is the probability that a player will roll a sum of 11 on his first roll of two dice? Express your answer as a common fraction.

Eleven is a palindrome. A palindrome is an integer that reads the same backward and forward. The integer 12,321 is a palindrome since writing the numbers in reverse order is also 12,321. What is the greatest possible four-digit palindrome that is a multiple of 6?