MATHCOUNTS® Problem of the Week Archive

The Mysteries of 11 – October 26, 2020

Problems & Solutions

As November (the 11th month) gets underway, it's the perfect time to focus on 11. Eleven is the fourth prime number, and there is a fun divisibility rule for 11. For any integer, insert alternating "–" and "+" signs between the consecutive pairs of digits, starting with a "–" sign between the left-most pair of digits. For example, for the number 91,828 we would have 9 – 1 + 8 – 2 + 8. (Notice that the first minus went between the left-most pair of numbers, 9 and 1, and then we alternated with "+" and "–" signs.) Now, simplify the expression. For our example, we have 9 – 1 + 8 – 2 + 8 = 22. Since this value, 22, is divisible by 11, the original number is divisible by 11. Using this rule, if the five-digit integer 76,9*a*2 is a multiple of 11, what is the value of *a*?

We can set up the expression 7 – 6 + 9 – a + 2 and simplify it to 12 – a. We now know that in order for 76,9a2 to be a multiple of 11, the expression 12 – a must be a multiple of 11. The multiples of 11 are 0, 11, 22, 33, etc. Since a must be a single digit, the only possibility is 12 – a = 11, and therefore, a = ***1****. We can check with a calculator to see that 76,912 is in fact divisible by 11.*

When playing many games, players must roll a pair of dice and find the sum of the two numbers rolled. With two dice, there are 11 possible sums ranging from 2 through 12. What is the probability that a player will roll a sum of 11 on his first roll of two dice? Express your answer as a common fraction.

Though there are 11 possible sums, it is much "harder" to roll a sum of 12 than a sum of 6. The 11 possible sums are not all equally likely. We could list out the different outcomes of a roll of two dice. (It might be easier to think of this as having a green die and a red die, so that we can clearly see that a roll of green-1 and red-3 is different than green-3 and red-1.) If we list out all of the outcomes, we see that there are 36 different possible roll outcomes resulting in sums from 2 to 12. (Alternatively, there are 6 possible rolls on each die, so rolling two dice could result in 6 × 6 = 36 different outcomes.) Of these 36 outcomes, only two of them result in a sum of 11. They are green-5 and red-6 and green-6 and red-5. Therefore, the probability of rolling a sum of 11 with two dice is 2/36 = ***1/18****.*

Eleven is a palindrome. A palindrome is an integer that reads the same backward and forward. The integer 12,321 is a palindrome since writing the numbers in reverse order is also 12,321. What is the greatest possible four-digit palindrome that is a multiple of 6?

The greatest four-digit palindrome is 9999. We need the four-digit palindrome to be a multiple of 6, which means that the units digit (and therefore the thousands digit) must be even. To make the number as large as possible, we need to start with 8_ _8. To be a multiple of 6, the number must be even (a multiple of 2), which we already have, and the number must also be a multiple of 3. A multiple of three has a digit-sum that is divisible by 3. Obviously, we'd like to have 8998, since that's the largest number we can now make. The digit-sum is 8 + 9 + 9 + 8 = 34, which is not divisible by 3, so 8998 is not our answer. We can test 8888, but this also does not work since the digit-sum is 32. Notice that our digit-sum

is decreasing by 2 when we decrease each of the missing digits by 1. When we get to 8778, we see that the digit-sum is 30, which is divisible by 3. Therefore, ***8778*** *is the largest four-digit palindrome divisible by 6.*

MATHCOUNTS® Problem of the Week Archive

The Mysteries of 11 – October 26, 2020

Problems

As November (the 11th month) gets underway, it's the perfect time to focus on 11. Eleven is the fourth prime number, and there is a fun divisibility rule for 11. For any integer, insert alternating "–" and "+" signs between the consecutive pairs of digits, starting with a "–" sign between the left-most pair of digits. For example, for the number 91,828 we would have 9 – 1 + 8 – 2 + 8. (Notice that the first minus went between the left-most pair of numbers, 9 and 1, and then we alternated with "+" and "–" signs.) Now, simplify the expression. For our example, we have 9 – 1 + 8 – 2 + 8 = 22. Since this value, 22, is divisible by 11, the original number is divisible by 11. Using this rule, if the five-digit integer 76,9a2 is a multiple of 11, what is the value of a?

When playing many games, players must roll a pair of dice and find the sum of the two numbers rolled. With two dice, there are 11 possible sums ranging from 2 through 12. What is the probability that a player will roll a sum of 11 on his first roll of two dice? Express your answer as a common fraction.

Eleven is a palindrome. A palindrome is an integer that reads the same backward and forward. The integer 12,321 is a palindrome since writing the numbers in reverse order is also 12,321. What is the greatest possible four-digit palindrome that is a multiple of 6?