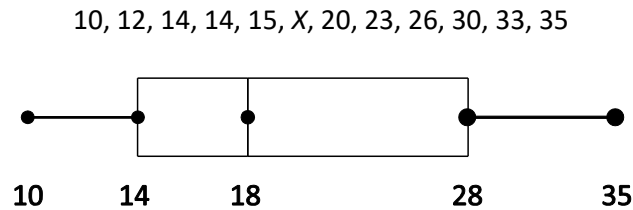


# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

## Stuck in the Middle – October 5, 2020

### Problems & Solutions

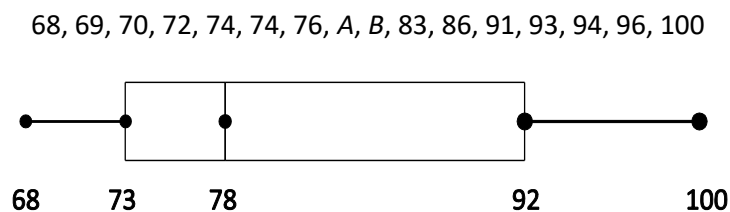
Below are 12 integer quiz scores listed in order from least to greatest followed by a box-and-whisker plot of the data. What is the value of  $X$ ?



According to the box-and-whisker plot, the median of the quiz scores is 18. Since there is an even number of scores, the median must be equidistant from the 6<sup>th</sup> and 7<sup>th</sup> quiz scores in the ordered list of 12 scores. In this case, 18 must be equidistant from  $X$  and 20. Since 20 is 2 more than 18, it follows that  $X$  must be 2 less than 18, which is **16**.

It is also true that the median would be the average of the 6<sup>th</sup> and 7<sup>th</sup> quiz scores in the ordered list. In this case, the median must be the average of  $X$  and 20. Solving algebraically, we have  $(X + 20)/2 = 18 \rightarrow X + 20 = 36 \rightarrow X = 16$ .

Below are 16 integer test scores listed in order from least to greatest followed by a box-and-whisker plot of the data. If 74 is the unique mode of the scores, what is the value of  $B$ ?



According to the box-and-whisker plot, the median of the test scores is 78. Since there is an even number of scores, the median must be equidistant from the 8<sup>th</sup> and 9<sup>th</sup> scores in the ordered list of 16 scores. In this case, 78 must be equidistant from  $A$  and  $B$ . One possibility is if  $A = 78$  and  $B = 78$ . But that means 78 is a mode, and we are told that the unique mode is 74. If we make  $A$  and  $B$  each a distance of 1 away from 78, we have  $A = 77$  and  $B = 79$ . We could also try making  $A$  and  $B$  each a distance of 2 away from 78. The result would be  $A = 76$  and  $B = 80$ . But we again have the issue of 76 being a mode, which cannot happen. If  $A$  and  $B$  are a distance greater than 2 away from 78, the list is no longer ordered. Therefore, it must be true that  $A = 77$  and  $B = 79$ .

What is the mean of the 16 test scores in the previous problem? Express your answer as a decimal to the nearest tenth.

*The mean of the scores is  $(68 + 69 + 70 + 72 + 74 + 74 + 76 + 77 + 79 + 83 + 86 + 91 + 93 + 94 + 96 + 100) \div 16 = 1302 \div 16 = 81.375 \approx \mathbf{81.4}$ .*

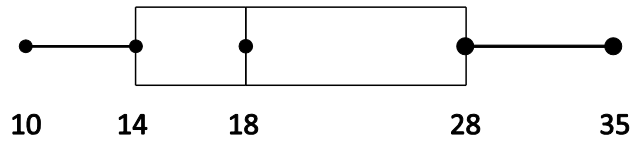
# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

## Stuck in the Middle – October 5, 2020

### Problems

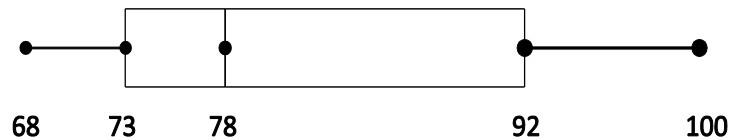
Below are 12 integer quiz scores listed in order from least to greatest followed by a box-and-whisker plot of the data. What is the value of  $X$ ?

10, 12, 14, 14, 15,  $X$ , 20, 23, 26, 30, 33, 35



Below are 16 integer test scores listed in order from least to greatest followed by a box-and-whisker plot of the data. If 74 is the unique mode of the scores, what is the value of  $B$ ?

68, 69, 70, 72, 74, 74, 76,  $A$ ,  $B$ , 83, 86, 91, 93, 94, 96, 100



What is the mean of the 16 test scores in the previous problem? Express your answer as a decimal to the nearest tenth.