MATHCOUNTS® Problem of the Week Archive

American Idol Mania! – September 9, 2019

Problems & Solutions

Season 17 of American Idol ended earlier this year. At the end of the season finale, each of the three judges was asked to share his/her opinion about who should win the contest; Laine or Alejandro. Each of the three judges responded and audiences around the country basically heard "Laine," "Laine," "Laine." If each judge answered either "Laine" or "Alejandro," how many additional ways could the judges' responses have been heard?

Each judge had the opportunity to vote for one of two people, so this is similar to a coin-flipping scenario where either Heads or Tails is possible. The first judge had two options (Laine or Alejandro). Then the second judge had two options, so the possibilities to that point would have been (Laine, Laine) or (Laine, Alejandro) or (Alejandro, Laine) or (Alejandro, Alejandro). Finally, the third judge's vote would add two possibilities to each of these four, for a total of 8 ways. A quick formula for this entire process is $2^3 = 8$. However, the question asked for the number of additional ways, other than the actual result, and that would be ***7****.*

Before hearing the results of the call-in voting on the season finale, viewers watch a couple of hours of songs from all ten of the finalists. This year, viewers may have noticed that there were conveniently 5 females and 5 males in the final group of ten. If a random group of ten distinct people is selected, and the probability of selecting a female for each individual selection is ½, what is the probability that this group of ten people will consist of 5 males and 5 females? Express your answer as a common fraction.

The first selection may be a Male (M) or Female (F). Then the second selection may be a Male or Female, so we already have the options MM, MF, FM, FF. This process is much like the process we used in counting the arrangements in the first problem. Therefore, the number of possible arrangements of Males and Females in a group of 10 people is $2^{10} = 1024$ arrangements. Now we need to figure out how many of these arrangements consist of exactly 5 males. To do this, we'll use the idea of combinations. How many different ways can we choose 5 people from a group of 10? Each of these ways is an arrangement with exactly 5 males. So, to calculate "10 choose 5", we will simplify $(10!) / (5! \times (10-5)!) = (10!) / (5! \times 5!) = 252$ ways. Therefore, 252 out of 1024 arrangements have exactly 5 males, which is a probability of 252/1024 = ***63/256****.*

It was reported that approximately 15.5 million telephone votes were cast. Each vote was for either Laine or Alejandro. If the ratio of "Laine votes" to "Alejandro votes" was 29:21, how many votes did Alejandro receive? What percent of the votes did Laine receive?

Since the ratio we are working with is 29:21, we can write the equation 29x + 21x = 15,500,000. Simplifying the left side yields 50x = 15,500,000. Therefore, by dividing both sides by 50, we get x = 310,000. So, Alejandro's number of votes was 21x = 21(310,000) = 6,510,000 votes. This means that Laine received 15,500,000 – 6,510,000 = 8,990,000 votes, which is 8,990,000 / 15,500,000 = ***58%*** *of the votes.*

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