MATHCOUNTS® Problem of the Week Archive

Abundant, Deficient or Perfect – July 13, 2020

Problems & Solutions

Whole numbers are said to be abundant, deficient, or perfect. A whole number is abundant if the sum of its proper divisors is greater than the whole number. The whole number 12 is abundant. Its proper divisors are 1, 2, 3, 4, and 6 and the sum of the proper divisors is 16 which is greater than 12. A whole number is deficient if the sum of its proper divisors is less than the whole number. The number 4 is deficient. Its proper divisors are 1 and 2 and the sum of its proper divisors is 3 which is less than 4. A whole number is perfect if the sum of its proper divisors is equal to the whole number. The number 6 is perfect. Its proper divisors are 1, 2, and 3 and the sum of its proper divisors is 6. This week's problems explore the results when abundant, deficient, and/or perfect numbers are combined in some way.

What is the least possible sum of two distinct abundant numbers that is not an abundant number?

Make a list of abundant numbers and check to see if the sum of two abundant numbers is abundant. The first few abundant numbers are: 12, 18, 20, 24, 30, 36, 40, … The least sum of two distinct abundant numbers is 12 + 18 = 30. Thirty is an abundant number. The next possible sum is 12 + 20 = 32. Thirty-two is not an abundant number. Its proper divisors are 1, 2, 4, 8 and 16, and their sum is 31. Therefore, ***32*** *is the least sum of two distinct abundant numbers that is not an abundant number.*

What is the least possible two-digit sum of two distinct deficient numbers that is not deficient?

Make a list of deficient numbers and check to see if the two-digit sum of two deficient numbers is deficient. The first few deficient numbers are 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, … The least two-digit sum of two deficient numbers is 2 + 8 = 10 or 3 + 7 = 10. Ten is a deficient number. The next possible two-digit sum is 2 + 9 = 11 or 3 + 8 = 11 or 4 + 7 = 11. Eleven is a deficient number. The next possible two-digit sum is 2 + 10 = 12 or 3 + 9 = 12 or 4 + 8 = 12 or 5 + 7 = 12. From above, we know that 12 is an abundant number, so ***12*** *is the least possible two-digit sum of two distinct deficient numbers that is not deficient.*

What is the least possible sum of an abundant number and a deficient number that is not abundant?

From our lists of abundant and deficient numbers, the least abundant number is 12 and the least deficient number is 2. Their sum is 12 + 2 = 14. Fourteen is not an abundant number, so ***14*** *is the least sum of an abundant number and a deficient number that is not abundant.*

What is the least possible abundant number that can be written as the sum of two distinct abundant numbers in exactly four ways?

*Use your list of abundant numbers and make a systematic list of the sums of two abundant numbers. You'll find that **66** is the least abundant number that can be expressed as the sum of two distinct abundant numbers in exactly four ways: 12 + 54, 18 + 48, 24 + 42 and 30 + 36.*

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