MATHCOUNTS® Problem of the Week Archive

Perfect Days in June – June 22, 2020

Problems & Solutions

The following problem of the week was submitted by Tim Ramsey of the Singapore American School in 1999.

Some would say that a day during late June would definitely be perfect. Halfway between the drenching showers of April and the brutal heat of August. To mathematicians, one particular day in June is perfect: June 28. When written in the form mm/dd, June 28 is written as 6/28, and both 6 and 28 are called *perfect numbers*. A perfect number is a number whose proper factors (that is, the factors other than the number itself) add up to the number. What are the proper factors of 6, and what is the sum of these proper factors?

*The proper factors of 6 are **1**, **2** and **3**. (Note that 6 is a factor of 6, but it is not a proper factor of 6.) The sum of the factors is 1 + 2 + 3 = **6**, which is why 6 is considered a perfect number.*

When was the last time that the day, month and year were all perfect numbers? When is the next time? (Hint: it may be easier to answer this question after you see the next question about Mersenne primes and perfect numbers.)

*After 6 and 28, perfect numbers get fairly scarce. The next two smallest perfect numbers are 496 and 8128. So, the last perfect date happened on **6/28/496**, and the next one will occur on **6/28/8128**.*

The formula for deriving perfect numbers is $(2^{p-1})(2^p - 1)$ where p and $(2^p - 1)$ are prime. If p and $(2p - 1)$ are prime, then $(2^p - 1)$ is called a *Mersenne Prime*. Find the first prime number that is not a Mersenne Prime.

*The sequence of prime numbers is 2, 3, 5, 7, 11, 13, …. The first prime number, **2**, is not a Mersenne prime, because it cannot be written in the form $(2^p - 1)$. Note, however, that 3 is a Mersenne prime, because $2^2 - 1 = 3$.*

The largest know Mersenne Prime is $2^{3021377} - 1$, and its associated perfect number has 909,526 digits. What is the units digit of this Mersenne Prime, and what is the units digit of the associated perfect number?

*The units digit of 2^1 is 2; of 2^2 is 4; of 2^3 is 8; of 2^4 is 6; of 2^5 is 2; of 2^6 is 4; and so on. Notice that the pattern of units digits is 2, 4, 8, 6, 2, 4, 8, 6 repeats every fourth term. Hence, the units digit of $2^{3021377}$ can be found by dividing the exponent by 4, noting the remainder r, and finding the units digit of 2^r. When 3,021,377 is divided by 4, the quotient is 755,344, remainder 1. Because the units digit of 2^1 is 2, the units digit of $2^{3021377}$ is also 2, and the units digit of $2^{3021377} - 1$ is therefore **1**. The associated perfect number is $(2^{3021377-1})(2^{3021377} - 1)$. The units digit of the first term, $2^{3021377-1}$, is 6. This follows from the fact that the units digit of $2^{3021377}$ was 2, and we need the units digit of one fewer power. And, we saw above that the units digit of $2^{3021377} - 1$ was 1. Hence, the units digit of this perfect number will be 6 × 1 = **6**.*

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