MATHCOUNTS® Problem of the Week Archive

A Tribute to Flag Day – June 15, 2020

Problems & Solutions

Yesterday, Sunday, June 14th, the United States celebrated Flag Day. If you check out the many websites devoted to Flag Day, you are sure to learn a lot about the history of the holiday and the history of the flag, the Pledge of Allegiance and the Star Spangled Banner. Francis Scott Key wrote the Star Spangled Banner in 1814 when he saw the enormous American flag (made by Mary Pickersgill) blowing in the wind approximately 8 miles away. Assuming the flag was visible to anyone within a 15,000-yard radius of the flag, how many square miles was the total area from which the flag could be viewed? Give your answer to the nearest whole square mile.

> *If anyone within a 15,000-yard radius can see the flag, then we are trying to find the area of a circle with a radius of 15,000 yards. Since we are going to have to change this into miles eventually, let's go ahead and do it now so that we don't have to work with such large numbers. There are 1760 yards in 1 mile, so dividing by 1760 gives us a circle with a radius of 8.522 miles. Therefore, the flag is visible to anyone within π(8.522)^2 = **228** square miles, to the nearest whole square mile.*

You've probably seen US flags in a variety of different sizes, but they are all geometrically similar and look the same because there are strict guidelines concerning the proportions of the different parts of the flag. A flag has a total of 13 stripes of equal width, starting with red at the top and alternating with white all the way down. There is also a rectangular region in the upper left-hand corner that is blue with white stars. Assuming that a flag is being displayed horizontally, all of the dimensions of the flag are compared to its width (top to bottom) measurement. Assume that the width is 1 unit, the length of the flag (right to left) is 1.9 units, the width of the blue rectangle is 7/13 of a unit (7 of the 13 stripes) and the length of the blue rectangle is .76 units long. Using these proportions, what percent, to the nearest whole number, of the surface area of the flag is red?

> *Looking at a flag while doing the problem will probably be helpful. Since we are working with the red parts of the flag, we are only concerned with the 7 red stripes. Remember that every stripe has a width of 1/13 units. However, notice 4 of the red stripes are shorter than the other three. The long three are rectangles that are 1.9 units by 1/13 units, so the total area of them is 3 × 1.9 × (1/13) = .438462 square units. The shorter 4 rectangles are (1.9 – .76) units by 1/13 units, so the total area of them is 4 × (1.9 – .76) × (1/13) = .350769 square units. Together, the 7 red stripes are .789231 square units. The area of the entire flag is 1 by 1.9, which gives an area of 1.9 square units. So, dividing .789231 ÷ 1.9 gives that red makes up **42%** of the U.S. flag, to the nearest whole percent.*

On many of the Flag Day websites, there are specific guidelines concerning the displaying of and care of a U.S. flag. Many people put flags out for certain holidays, Flag Day being one of the most popular. Suppose you have 48 little flags each attached to 3-foot stakes and want to place them around the perimeter of your garden in the front yard. Your garden is a perfect square measuring 8 yards by 8 yards and you want to equally space the flags around its perimeter. If you place the first flag in the front, right

corner and continue placing them around the entire garden, how many feet apart do you have to place them and how many flags will be along the back side of the garden?

*First, the perimeter of the garden is 4 × 8 = 32 yards. If there are 48 flags, then we need to take 32 and divide by 48, giving us 2/3 yards or 2 feet. So, we need to put a flag every two feet starting at the front right corner. To find the number of flags per side of the garden, it may appear that you just have to divide 48 by 4 and see that there are 12 flags per side, but the flags in the corners make a difference. Think of it as we are starting at the front right corner and putting in the corner flag along with 11 other flags on the first side. Moving to the second side, we are also starting with the corner and placing 11 more flags along that side. Notice that the corner flag we started the second side with is also on the first side. So, each side of the garden actually has **13** flags.*

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