

MATHCOUNTS[®] Problem of the Week Archive

The Unofficial Start of Summer – May 25, 2020

Problems & Solutions

Memorial Day has always been the unofficial start of summer. Here are some problems about some things we love in hot weather!

Two local supermarkets sell watermelons grown at McDowell's farm. But Biggy Market sells their watermelons for \$3.99 each, regardless of size, whereas Harmless Trail Market sells their watermelons for \$0.19 per pound. For what weight will a watermelon at Harmless Trail cost the same as any watermelon at Biggy? If the largest watermelon grown at McDowell's farm weighs 18 pounds, at which place should you buy your watermelons?

*We can rewrite \$3.99 as 399 cents. So, we want to know the number of pounds, w , for which $19w = 399$. Dividing by 19 on both sides of the equation gives $w = 21$ pounds as the weight at which a watermelon will cost the same at both supermarkets. If the largest watermelon grown at McDowell's farm weighs 18 pounds, then you should buy your watermelons at **Harmless Trail Market**.*

The SPF (sun protection factor) listed on sunscreen is an indication of how long you will be protected from sunburn when wearing the sunscreen. The amount of time you're protected is proportional to the SPF. If wearing SPF 8 sunscreen will protect you for 40 minutes, how long will SPF 30 sunscreen protect you?

*Because the time protected is proportional to the SPF, the equation $8/40 = 30/t$ represents the scenario. Cross multiplying gives $8t = 1200$, so $t = 150$ minutes. So, SPF 30 sunscreen will protect you for **2 hours, 30 minutes**.*

A soda manufacturer knows that sales of soda increase during the summer. To make sure that they get a large portion of the sales, they decide to offer a contest throughout the spring. The contest is pretty simple: when you open your bottle of soda, if it says, "You win!" under the cap, you get another bottle of their soda for free. The chance of winning a free soda is 1 in 12. Lucky Lucy bought five sodas and won a free one with four of the caps. What's the probability of that happening? Theoretically, how many sodas would you have to buy before having a greater than 50% chance of winning a free soda?

The probability of winning is $1/12$, and the probability of not winning is $11/12$. So, the probability of winning exactly four times in five tries is $(1/12)^4(11/12) = 11/248,832$, or speaking technically, highly unlikely.

*There is an $11/12$ chance of not winning on each bottle. So, the probability of not winning when purchasing n bottles is $(11/12)^n$. We need to find the number of n for which $(11/12)^n$ is less than 50%, because then the probability of winning (1 minus that value) will be greater than 50%. Using a calculator and some guess and check work, it can be found that $(11/12)^n$ is less than 0.50 when $n > 7$, so a minimum of **8 bottles** need to be purchased to have a greater than 50% chance of winning.*

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