

MATHCOUNTS[®] Problem of the Week Archive

Spring Growth – May 4, 2020

Problems & Solutions

A 6-inch tall flower casts a 4-inch shadow at 5:00pm. One month later, when the sun was at the same position in the sky as it was at 5:00pm one month earlier, the shadow cast was $5\frac{1}{3}$ inches long. How many inches taller was the flower when it cast the $5\frac{1}{3}$ -inch shadow than it was when it cast the 4-inch shadow?

We can set up the following proportion to represent the scenario: $6/x = 4/(5\frac{1}{3})$. Cross multiplying gives $4x = 32$, so $x = 8$ inches. Thus, the flower was $8 - 6 = 2$ inches taller.

Two years ago Melanie planted two trees. Tree A was 24 inches tall when she planted it and Tree B was 40 inches tall when she planted it. If Tree A grows at a rate of 8 inches per year and Tree B grows at a rate of 6 inches per year, after how many more years will Tree A be as tall as Tree B?

Each year, Tree A's height gets 2 inches closer to Tree B's height. Since there is a difference of $40 - 24 = 16$ inches, it will take $16/2 = 8$ years from the time they were planted to be equal height. Eight is not our answer though. Since the trees were planted two years ago, only another $8 - 2 = 6$ years will have to pass before the two trees are equal height.

Last spring, Fredrico's favorite topiary bush formed a perfect sphere with a diameter of 16 inches. Over the past year the bush's diameter has increased by one inch. By what percent has the volume of the bush's sphere increased? Express your answer to the nearest tenth.

Last spring, Fredrico's bush had a volume of $(4/3)(\pi)(8^3) = 2144.660585$ cubic inches.

This year, Fredrico's bush has a volume of $(4/3)(\pi)(8.5^3) = 2572.440785$ cubic inches.

Thus, the percent of increase in the volume of the bush is $[(2572.440785 - 2144.660585)/2144.660585] \times 100 = 19.9\%$, to the nearest tenth.

Notice, too, our answer can be calculated with $[(4/3)(\pi)(8.5^3) - (4/3)(\pi)(8^3)]/[(4/3)(\pi)(8^3)]$. When $(4/3)\pi$ is factored out of the numerator and denominator, this simplifies to $[(8.5^3 - 8^3)/(8^3)] \times 100 \approx 19.9\%$.

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