MATHCOUNTS® Problem of the Week Archive

State Competitions – February 24, 2020

Problems & Solutions

State Competitions are coming up fast! Are you ready to compete? Let's try a few 2019 State Competition problems to get ready.

2019 State Sprint Round, #18

If C is a digit such that the product of the three-digit numbers 2C8 and 3C1 is the five-digit number 90C58, what is the value of C?

Let's work with just the rightmost two digits. For the units digit, 8 × 1 = 8 does not impact the tens digit. To get the tens digit of the product, we need to cross-multiply the units and tens digits of the two factors: C × 1 + 8 × C = 9C must end in 5. The only digit for C for which that works is C = ***5****.*

2019 State Target Round, #7

Andy has a cube of edge length 10 cm. He paints the outside of the cube red and then divides the cube into smaller cubes, each of edge length 1 cm. Andy randomly chooses one of the unit cubes and rolls it on a table. If the cube lands so that an unpainted face is on the bottom, touching the table, what is the probability that the entire cube is unpainted? Express your answer as a common fraction.

When a cube is subdivided along each of the three face-centered axes into n congruent slabs, a block composed of n^3 congruent smaller cubes is formed. Each of those smaller cubes has 6 faces, resulting in a total of $6n^3$ faces. Only the outer surface – the 6 faces – of the original cube is painted. Each of the 6 faces of the original cube involves 1 face from each of the n^2 smaller cubes making up the larger face, yielding $6n^2$ smaller faces that are painted, with the remaining $6n^3 – 6n^2$ smaller faces unpainted. Removing the outer layer on each face yields an $(n – 2) × (n – 2) × (n – 2)$ cube of totally unpainted smaller blocks, with $6(n – 2)^3$ unpainted smaller faces. Thus, with each of the $6n^3 – 6n^2 = 6(n^3 – n^2)$ unpainted smaller faces equally likely, of which $6(n – 2)^3$ correspond to completely unpainted smaller cubes, the probability of landing on a completely unpainted small cube upon landing on an unpainted smaller face is $\frac{(n-2)^3}{n^3-n^2}$*. When n = 8, the probability is* $\frac{6^3}{8^3-8^2} = \frac{216}{512-64} = \frac{216}{448} = \mathbf{\frac{27}{56}}$.

2019 State Team Round, #4

Suppose that Martians have eight fingers and use a base-eight (octal) number system. If Marty the Martian says he is 37 years old on Mars, how old is he in Earth's base-ten system?

*Just as 37 as a base-ten number means $3 \times 10^1 + 7 \times 10^0 = 3 \times 10 + 7 \times 1 = 37$, so 37 as a base-eight number means $3 \times 8^1 + 7 \times 8^0 = 3 \times 8 + 7 \times 1 = 24 + 7 =$ **31 years** in base ten.*

2019 State Countdown Round, #12

For a particular sequence, each term is the sum of the three preceding terms. If a, b, c, d, e, 0, 1, 2, 3 are consecutive terms of this sequence, what is the value of $a + b + c + d + e$?

*As we're told, each term is the sum of the three preceding terms. In order for this to be true, $2 = 1 + 0 + e$, so e must equal 1. Similarly, $1 = 0 + e + d = 0 + 1 + d$, so $d = 0$. Then, $0 = e + d + c = 1 + 0 + c$, so $c = -1$. Continuing this pattern, we find that $b = 2$ and $a = -1$. Therefore, $a + b + c + d + e = -1 + 2 + -1 + 0 + 1 =$ **1**.*

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