

# MATHCOUNTS® Problem of the Week Archive

## Lunar New Year – February 8, 2021

### Problems & Solutions

While many countries celebrate Lunar New Year, the specific date and set of 12 zodiac animals varies. The problems below reflect the date and animals used in China, and Lunar New Year is referred to as “Chinese New Year” to reflect this distinction.

The modern (Babylonian) zodiac consists of 12 astrological signs corresponding to 12 different date ranges spanning two months each. For example, the zodiac sign Capricorn corresponds to the date range December 22<sup>nd</sup> to January 20<sup>th</sup>.

The Chinese zodiac consists of 12 animal signs, and each year corresponds to a different animal sign, and the cycle of animal signs repeats every 12 years. January 23, 2012 began a year of the Dragon. The 12 animal signs in order are Dragon, Snake, Horse, Goat, Monkey, Rooster, Dog, Pig, Rat, Ox, Tiger and Rabbit. The next time the Chinese New Year begins on January 23<sup>rd</sup> is in the year 2031. What animal sign corresponds to the year 2031?

*Since the cycle is 12 years, the next year of the Dragon will be in 2024. The year 2031 is seven years after 2024. The animal sign that occurs seven years after the Dragon is the **Pig**.*

Traditionally, each year is also associated with one of the five elements of Chinese astrology, which, in order, are Wood, Fire, Earth, Metal and Water. This association changes every two years with every other year alternating between Yin and Yang. For example, 2011 was the year of the Yin Metal Rabbit, 2012 was the year of the Yang Water Dragon and 2013 was the year of the Yin Water Snake. The next year of the Yang Water Dragon will occur in what year?

*From the previous problem, we know that the animal signs are on a 12-year cycle. The five elements occur twice every 10 years, and the Yin-Yang cycle is every other year. We can determine the LCM of 12, 10 and 2 to see how often each combination of animal, element and Yin-Yang will occur. The LCM of 12, 10 and 2 is 60, so the next Yang Water Dragon year will occur 60 years from the last Yang Water Dragon year (2012). So, it will occur in  $2012 + 60 = 2072$ .*

Suppose a Mathematical calendar was established in 2012 as another way to classify each year. Since 2012, the Mathematical New Year begins each year on March 14<sup>th</sup>. Each year is associated with an operator, a branch and a color. In order, the four operators are Addition, Subtraction, Multiplication and Division. There are seven branches. In order, they are Algebra, Geometry, Trigonometry, Calculus, Combinatorics, Probability and Statistics. If the cycle of operators repeats every four years and the branches repeat every seven years, what is the minimum number of colors needed for each yearly operator-branch-color designation to repeat every 140 years?

*As with the previous problem, we can use the LCM. The cycle of operator-branch-color designation is to repeat every 140 years, which means the LCM of the number of operators, the number of branches and the number of colors is 140. We are told that there are four operators and seven branches, so each*

*operator-branch pairing will repeat every  $7 \times 4 = 28$  years. If we divide 140 by 28, we see that the minimum number of colors needed is  $140 \div 28 = 5$  colors.*

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### ***Problems***

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