MATHCOUNTS® Problem of the Week Archive

Let the Competitions Begin! – February 3, 2020

Problems & Solutions

Chapter Competitions officially started this past weekend! Are you ready to compete? Let's try a few 2019 Chapter Competition problems to get warmed up.

<u>2019 Chapter Sprint Round, #20</u>

Jones is chasing a car 800 meters ahead of him. He is on a horse moving at 50 km/h. If Jones catches up to the car in 4 minutes, how fast was the car moving?

Jones catches up 800 m in 4 min and thus, at a rate of:
800m / 4min = 200 m / 1 min
200 m/min × (1 km / 1000m) × (60 min / 1h) = 12 km/h.
Jones is going 50 km/h, so the car is going 50 – 12 km/h = ***38 km/h****.*

<u>2019 Chapter Target Round, #6</u>

Lisa wants to use her calculator to square a two-digit positive integer, but she accidentally enters the tens digit incorrectly. When she squares the number entered, the result is 2340 greater than the result she would have gotten had she correctly entered the tens digit. What is the sum of the two-digit number Lisa entered and the two-digit number she meant to enter?

Let the desired tens digit be d, the wrongly input tens digit be w, and the units digit be u. Then the desired number to square is 10d + u and the wrongly squared number is 10w + u. The difference in their squares is $2340 = (10w + u)^2 – (10d + u)^2 = (100w^2 + 20wu + u^2) – (100d^2 + 20du + u^2) = 100(w^2 – d^2) + 20(w – d)u$*. Dividing both sides by 20 yields:* $117 = 5(w^2 – d^2) + (w – d)u = [5(w + d) + u](w – d)$*. We are dealing with tens digits d and w of 2-digit numbers, with w > d, so we must have* $1 \le d < w \le 9$*. Thus,* $1 \le w – d \le 8$ *and w – d must divide* $117 = 3^2 \times 13$*, meaning that w – d is 1 or 3. If w – d = 1, then 5(w + d) + u = 117, but w ≤ 9, d ≤ 8 and u ≤ 9, so 5(w + d) + u ≤ 94 and cannot be 117. Therefore, w – d = 3. Now we have 117 = [5(w + d) + u](3), so 5(w + d) + u = 39, which means: w + d = 6 and u = 9, or w + d = 7 and u = 4. Since w – d is odd (3), w + d must likewise be odd. Thus, w + d = 7 and u = 4 holds. Now, we are to find (10w + u) + (10d + u) = 10(w + d) + 2u = 10 × 7 + 2 × 4 =* ***78****.*

2019 Chapter Team Round, #9

Jane has six different hamsters for which she has two cages, one red and one blue. She wants to put three hamsters in the red cage and three in the blue cage, but two of the hamsters, Felix and Oscar, do not get along and cannot be in the same cage. In how many different ways can she choose which three hamsters to put in the red cage?

Either Felix or Oscar goes in the red cage; the other goes in the blue cage – this yields 2 options. For each of those two options, there are 4 remaining hamsters. Any of the 4 may be second to occupy the red cage, and after that choice is made, any of the 3 remaining ones may be third to occupy the red cage. Now, the last 2 go in the blue cage. Thus, it appears that there are $2 \times 4 \times 3 = 24$ ways of selecting which hamsters will occupy the red cage. However, we could have swapped which hamster was chosen second versus which hamster was chosen third, and we would end up with the same occupancy in each cage. Therefore, we need to divide by 2 to avoid double counting, so we end up with 24 / 2 = ***12 ways****. We could also have done it as 2 (for Felix and Oscar) times the number of ways (combinations) to distribute 4 things (the remaining 4 hamsters) taken 2 at a time (2 to go in the red box) to get $2 \times [4! / (2!2!)] = 2 \times [24 / (2 \times 2)] =$* ***12 ways****.*

2019 Chapter Countdown Round, #53

The sum of the values of *a* and *b* is 47. The sum of the values of *a* and *c* is 126. The sum of the values of *b* and *c* is 93. What is the value of *a*?

We can write the provided information as equations:

$a + b = 47$

$a + c = 126$

$b + c = 93$

Let's start with the first equation. If we know $a + b = 47$, then by subtracting a from both sides of this equation, we find that $b = 47 - a$. We can substitute this into the third equation for b, to get $(47 - a) + c = 93$. Subtracting 47 from both sides of this equation gives us $-a + c = 46$, and adding a to both sides of this equation gives us $c = 46 + a$. Finally, we can substitute this into the second equation for c to find that $a + (46 + a) = 126$. Combining like terms and subtracting 46 from both sides of this equation gives $2a = 80$. Finally, dividing by 2 on both sides of this equation gives us that ***a = 40****.*

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