MATHCOUNTS® Problem of the Week Archive

Best of 2018 – December 31, 2018

Problems & Solutions

As 2018 comes to an end, let's go back and solve some of our favorite problems of year.

School Handbook

Problem 237: Kendra starts at a positive integer k and counts up by 4s until she hits exactly 200. Mason starts at a positive integer m and counts up by 6s until he hits exactly 200. If it takes Kendra half as many steps to reach 200 as it takes Mason, what is the greatest possible value of $k - m$?

> *To get the greatest possible value of $k - m$, we want to make m as small as possible. Working backward, we can subtract 6 from 200 a total of 33 times and get 2 (because 200 − 33 × 6 = 2). If we continue to subtract 6, the result is a negative number. But Mason must take an even number of steps, since Kendra takes only half as many steps. Let's have Mason start at 8 and count up by 6 a total of 32 times. Then Kendra would have to start at 200 − 16 × 4 = 136. This gives us the greatest possible difference k − m = 136 − 8 = **128**.*

School Competition

Sprint 23: A bag contains 25 tickets, each colored either red or yellow. Red tickets are worth $0.50, and yellow tickets are worth $5.00. If the expected value of a ticket drawn at random from this bag is $3.20, how many of the tickets are red?

> *Let r and y represent the number of red and yellow tickets, respectively. From the given information, we can write the following equations: r + y = 25 and (r/25) × (0.5) + (y/25) × (5) = 3.20. Solving the first equation for y gives us y = 25 − r. Substituting 25 − r for y in the second equation and solving for r yields (r/25) × (0.5) + ((25 − r)/25) × (5) = 3.20 → 0.5r/25 + (125 − 5r)/25 = 3.20 → (125 − 4.5r)/25 = 3.20 → 125 − 4.5r = 80 → −4.5r = −45 → r = **10** tickets.*

Chapter Competition

Target 5: Aiden and Bryce are racing around a race track. They begin together at the starting line, and Aiden's car completes a lap every 44 seconds, while Bryce's car completes a lap every 40 seconds. How many seconds after they begin the race will Aiden and Bryce first reach the starting line at the same time?

> *Let a and b be the number of laps that Aiden and Bryce, respectively, make when they meet again at the starting line. Because the requisite meeting place is at the starting line, that involves a whole number of laps for each, thus making both a and b to be integers. The times involved are 44a seconds and 40b seconds, which must be equal to be a meeting. Now, 44a seconds = 40b seconds, and manipulating algebraically yields b/a = 44/40 = 11/10, which cannot be reduced further. Therefore, the first time they reach the starting line at the same time occurs when Bryce has completed 11 laps and Aiden has completed 10 laps, which happens 10 × 44 = 11 × 40 = **440** seconds after the race begins.*

National Competition

Winning Countdown Question: The first three terms of an infinite arithmetic sequence are 3.46, 2.47 and 1.48, in that order. What is the first integer term in this sequence?

> *From the first three terms of this arithmetic sequence, we see that each term is 0.99 less than the previous term. Using this to determine the next few terms of this sequence we get 3.46, 2.47, 1.48, 0.49, −0.50, −1.49, −2.48, …. If we just look at the negative terms in the sequence, we see a pattern in both the digits before the decimal point (0, 1, 2, …) and in the digits after the decimal point (50, 49, 48, …). The first integer term of the sequence will have only 0s after the decimal point. So, continuing the pattern of the digits after the decimal point (50, 49, 48, 47, …, 03, 02, 01, 00), we see that the 51st negative term of the sequence will have only 0s after the decimal point. Continuing the pattern of the digits before the decimal point (0, 1, 2, 3, …, 47, 48, 49, 50), we see that the 51st negative term will have the digits 50 before the decimal point. Therefore, the first integer term of the sequence, which happens to be the 51st negative term of the sequence, has the value **−50**.*

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