

# MATHCOUNTS® Problem of the Week Archive

**Best of 2017 – December 25, 2017**

## Problems & Solutions

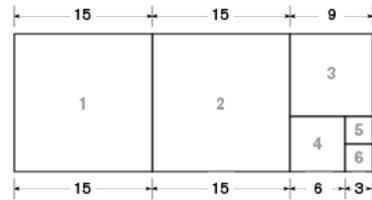
As the year comes to a close, let's take a look back at some of our favorite problems of 2015.

### 2017 School Sprint #29

Day	1	2	3	4	5	6
Eaten Area (cm <sup>2</sup> )	225	225	81	36	9	9

Coco buys a very large rectangular chocolate bar and decides that each day, she will cut the largest possible square off of the bar and eat it. When the remaining part of the chocolate bar is a square, she will eat all that is left. The table shows the area, in square centimeters, of the square Coco eats on each day. If Coco finishes the chocolate bar on Day 6, what was the length of the longer side of the chocolate bar when Coco bought it?

On each of the six days Coco ate a square piece of chocolate with side length 15 cm, 15 cm, 9 cm, 6 cm, 3 cm and 3 cm, respectively. Since we know the originally chocolate bar was a rectangle the 6 squares must fit together as shown. The length of the longer side of the original chocolate bar, then, was  $15 + 15 + 9 = 39$  cm.



### 2017 State Target #4

+	$a$	$b$	$c$	$d$
$a$	$n+10$			$n-2$
$b$				
$c$			$n+6$	
$d$		$n+3$		

The addition table shown has rows and columns labeled with integers  $a$ ,  $b$ ,  $c$  and  $d$ , in that order. A few of the sums in the table are already filled in; for example, the table shows that  $a + d = n - 2$ . When all sixteen sums are filled in, the sum of the sixteen entries in the table is  $16n + k$ , where  $k$  is an integer. What is the value of  $k$ ?

The sum of the contents of all 16 cells includes 8 copies of each of  $a$ ,  $b$ ,  $c$  and  $d$ . For example, for the  $a$ 's the cells with " $a$ " are those in the " $a$ " row and those in the " $a$ " column (being careful not to double-count the cell where the  $a$  row and column meet), so  $(a + a) + (a + b) + (a + c) + (a + d) + (b + a) + (c + a) + (d + a)$  contains 8 copies of  $a$  being added. Therefore, the sum of all the cells is  $8a + 8b + 8c + 8d = 8a + 8c + 8(b + d)$ . Now,  $a + a = 2a = n + 10$ , so  $8a = 4n + 40$ ;  $c + c = 2c = n + 6$  so  $8c = 4n + 24$ ;  $b + d = n + 3$  so  $8(b + d) = 8n + 24$ . Therefore, the sum of all the cells is  $8a + 8c + 8(b + d) = 4n + 40 + 4n + 24 + 8n + 24 = 16n + 88$ , so  $k = 88$ .

### 2017 National Countdown Winning Question

In a barn, 100 chicks sit peacefully in a circle. Suddenly, each chick randomly pecks the chick immediately to its left or right. What is the expected number of un-pecked chicks?

*If we look at any one particular chick, there is a 1 in 2 chance that it will not be pecked by the chick to its left and a 1 in 2 chance it will not be pecked by the chick to its right. This means there is a 1 in 4 chance it will not be pecked at all. This probability is the same for every chick in the circle. Since there are 100 chicks total, we can expect 1 in 4 to be un-pecked or **25** chicks.*

*Alternatively, for any one chick, there are four possible outcomes. Being pecked only by the chick to its left, being pecked only by the chick to its right, being pecked by both the chicks or being pecked by neither of the chicks. Each of the four outcomes is equally probable, so there is a 1 in 4 chance of any chick being un-pecked. Since there are 100 chicks in the circle we can expect,  $1/4 \times 100 = \mathbf{25}$  un-pecked.*

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