

MATHCOUNTS® Problem of the Week Archive

Happy Halloween – October 29, 2018

Problems & Solutions

Mrs. Miller bought a half bushel of apples, 16 of which were set aside to be used for apple bobbing at the harvest festival. That left only two-thirds of the apples Mrs. Miller bought. She then used three-fourths of the remaining apples to make four apple pies. If Mrs. Miller was left with just enough apples to make two caramel apples for each of her children, how many children does Mrs. Miller have?

Let n represent the total number of apples in the half bushel that Mrs. Miller bought. The 16 apples set aside to be used for bobbing represent $1/3$ of the total number of apples, so $1/3n = 16 \rightarrow n = 48$ apples. That leaves $48 - 16 = 32$ apples. The apple pies are made using $3/4$ of the remaining apples, or $3/4(32) = 24$ apples, leaving $32 - 24 = 8$ apples. Mrs. Miller uses the 8 apples to make each of her children two caramel apples, so Mrs. Miller must have $8 \div 2 = 4$ children.

In two hours of trick-or-treating, Reese, Clark and Joy filled six bags with treats. At this rate, how many bags of treats could Joy fill alone, in one hour of trick-or-treating?

*If the three friends fill six bags in two hours, then, together, they fill three bags per hour. Since each person does $1/3$ of the work, it follows that, working alone, Joy could fill **one** bag in one hour of trick-or-treating.*

A house was haunted by a combined total of 51 ghosts, goblins and ghouls. On Friday, there were half as many ghosts as there were goblins. On Saturday, two-thirds of the ghouls each became a ghost. On Sunday, 11 of the ghosts each became a goblin, and the ratio of ghouls to goblins became 1:3. If no other changes occurred, how many ghosts are there?

If there were x ghosts in the house on Friday, and there were half as many ghosts as goblins, or twice as many goblins as ghosts, then there were $2x$ goblins in the house on Friday. Let's say there were y ghouls in the house on Friday. We also know that $x + 2x + y = 51 \rightarrow 3x + y = 51$. On Saturday, $\frac{2}{3}y$ ghouls became ghosts, so there were $x + \frac{2}{3}y$ ghosts, $\frac{1}{3}y$ ghouls and still $2x$ goblins. On Sunday, 11 ghosts became goblins so there are now $x + \frac{2}{3}y - 11$ ghosts, $2x + 11$ goblins and $\frac{1}{3}y$ ghouls. Since the ratio of ghouls to goblins is 1:3, we can write the proportion $\frac{\frac{1}{3}y}{2x+11} = \frac{1}{3}$. Cross-multiplying, we get the equation $y = 2x + 11$. Substituting for y in the previous expressions and simplifying, we see $x + \frac{2}{3}y - 11$ ghosts = $\frac{1}{3}(7x - 11)$ ghosts, and $\frac{1}{3}y$ ghouls = $\frac{1}{3}(2x + 11)$ ghouls. Then, substituting for y in the equation $3x + y = 51$ yields $3x + 2x + 11 = 51 \rightarrow 5x + 11 = 51 \rightarrow 5x = 40 \rightarrow x = 8$. So, there are now $\frac{1}{3}(7x - 11) = \frac{1}{3}(7(8) - 11) = \frac{1}{3}(56 - 11) = \frac{1}{3}(45) = 15$ ghosts.

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