**Problem of the Week Archive**

**Asteroid Blaster – September 10, 2018**

**Problems & Solutions**

In the video arcade game Asteroid Blaster there are a total of five rounds on the first level. During each round, asteroids appear on the screen and players must blast as many as possible before time expires. To advance to the next round, a player must destroy all the asteroids in the current round. This becomes more difficult to accomplish because the number of asteroids increases with each round. Rounds 1, 2 and 3 each have 1, 8 and 27 asteroids, respectively. If this pattern continues, how many asteroids will there be to destroy in Round 4?

We can't really tell much by just looking at Round 1. But for Round 2 we can see there are 8 asteroids. The pattern could be $2 \times 4 = 8$ or $2 \times 2 \times 2 = 8$. Let's look at Round 3 to get more information. Round 3 has 27 asteroids, which is either $3 \times 9$ or $3 \times 3 \times 3$. It looks like the pattern is 13, 23, 33, … In which case, the next round would have $43 = 4 \times 4 \times 4 = 64$ asteroids.

As the number of asteroids increases with each round of Asteroid Blaster, so does the amount of time within which a player must blast all the asteroids. The time limit, in seconds, for each of the first four rounds is 2, 4, 8 and 16, respectively. If this pattern continues, what is the maximum number of seconds allotted to complete all five rounds of this level?

In the first round there are 2 seconds. The second round has a time limit of $2 \times 2 = 4$ seconds. Then the third round gives the player $2 \times 2 \times 2 = 8$ seconds. In Round 4 a player has $2 \times 2 \times 2 \times 2 = 16$ seconds. Assuming this pattern continues, a player will have $2 \times 2 \times 2 \times 2 \times 2 = 32$ seconds to complete Round 5. Therefore, the maximum allotted time to complete all five rounds of this level is $2 + 4 + 8 + 16 + 32 = 62$ seconds.

The number of points awarded for blasting each asteroid also increases with each round. The number of points awarded for each asteroid shot in Rounds 1 through 5 is 10, 20, 30, 40 and 50, respectively. How many points will a player earn for completing Rounds 1 through 4 and blasting 2/5 of the asteroids in Round 5 before time expires?

Based on the information from the first problem, we know that a player will earn $(1 \times 10) + (8 \times 20) + (27 \times 30) + (64 \times 40) = 10 + 160 + 810 + 2560 = 3540$ points for blasting every asteroid in Rounds 1 through 4. Assuming the total number of asteroids in Round 5 is $5 \times 5 \times 5 = 125$ asteroids, then $2/5$ of the asteroids in that round that is $2/5(125) = 50$ asteroids. From the previous problem we can assume that 50 points are awarded for each asteroid blasted in Round 5 for a total of $50 \times 50 = 2500$ points. Therefore, the total number of points the player will earn is $3540 + 2500 = 6040$ points.
Problems

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