

MATHCOUNTS® Problem of the Week Archive

Happy New (School) Year! – August 27, 2018

Problems & Solutions

A new school year often means new school clothes, new teachers and new friends. One of our favorite parts of a new school year is the release of a new *MATHCOUNTS School Handbook!* Download your free copy after solving a few problems from some previous handbooks.

Lily is going to the movies with Abby, Bea and Jaclyn. Abby wants to sit at the end of a row, and Bea only cares that she is seated next to Jaclyn. In how many different ways can the girls be seated in a single row that has only four seats?

There are 2 possible ends for Abby (A), 2 ways that Bea (B) and Jaclyn (J) can be next to each other and 2 choices for Lily (L) – either on the other end or between Abby and the pair of girls, Bea and Jaclyn. Here are the $2 \times 2 \times 2 = 8$ ways they all can be seated: A B J L, A J B L, A L B J, A L J B, B J L A, J B L A, L B J A, L J B A.

Harvey has a fair eight-sided die that has a different number from 1 to 8 on each side. If he rolls this die twice, what is the probability that the second number rolled is greater than or equal to the first number? Express your answer as a common fraction.

There are $8 \times 8 = 64$ ways the two rolls of the eight-sided die can occur. This will be the denominator of our probability fraction. If the first roll of the die is a 1, then the second roll can be any of the 8 numbers for it to be greater than or equal to the first roll. If the first roll is a 2, then the second roll can be any of 7 possibilities, excluding 1. With a 3, we can have 6 possibilities, etc. The total number of acceptable rolls for the second die is, thus, $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$. The probability is $36/64 = 9/16$.

Zeus threw, on average, 12 lightning bolts per day in the month of March. During the first week of April, he averaged 15 lightning bolts per day. How many lightning bolts does Zeus need to throw per day on average for the rest of April to maintain a 12-bolt-per-day average over March and April? Express your answer to the nearest whole number.

April has 30 days, so Zeus needs to throw a total of $30 \times 12 = 360$ lightning bolts in all of April. So far, he has thrown $7 \times 15 = 105$ lightning bolts. In the next $30 - 7 = 23$ days, he needs to throw $360 - 105 = 255$ lightning bolts. That's an average of about $255 \div 23 \approx 11$ lightning bolts per day.

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