

MATHCOUNTS® Problem of the Week Archive

Binary Math – July 31, 2017

Problems & Solutions

Binary math is math in base 2, instead of our typical base 10. All digits are 0s or 1s. Explore base 2 math in the following problems. The last one is a challenge, but if you get it, then you'll have a trick for guessing numbers!

What is the value of 31 in base 2?

Similar to how in base 10 we write 31 because $31 = 3 \times 10^1 + 1 \times 10^0$, 31 can be written as the expression: $31 = 16 + 8 + 4 + 2 + 1 = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$. So, the base 2 representation of 31 is **11111**.

What is the value of the following base 2 expression: $1101 + 10011$? Express your answer in base 2 and base 10.

Like addition in base 10 where if digits sum to 10 or greater you write that digit and carry the 1 to the next column, in base 2 if digits sum to 2 or greater you carry the 1. The value of the arithmetic problem in base 2 is:

$$\begin{array}{r} 1101 \\ +11011 \\ \hline 100000 \end{array}$$

This is $1 \times 2^5 = 1 \times 32 = 32$. So, our sum is **100000** base 2 and **32** base 10.

Marshall tells Karen that he can guess any number she thinks of between 1 and 1000 in ten guesses, or less. After each guess, Karen needs to say whether Marshall's guess is correct, or whether her number is higher than his guess or lower than his guess. Karen thinks of a number, and Marshall begins to guess her number as follows:

Marshall: Is it 512?

Karen: Higher.

Marshall: Is it 768?

Karen: Lower.

Marshall: Is it 640?

Karen: Higher.

Marshall: Is it 704?

Karen: Lower.

Marshall: Is it 672?

Karen: Higher.

Continuing to follow this strategy, Marshall correctly guesses Karen's number on his next guess. What number was Karen thinking of?

Let's look at Marshall's guesses to identify his process, keeping in mind that we know it has something to do with binary math. His first guess is 512, which is 2^9 . He then guesses 768, which is $768 - 512 = 256 = 2^8$ higher than his previous guess. Next, he guesses 640, which is $768 - 640 = 128 = 2^7$ lower than his guess. We now see the pattern, each guess he adds or subtracts, depending on Karen's response, the next lowest power of 2. When he guesses 704, he adds 2^6 , and when he guesses 672, he subtracts 2^5 . Since Karen says higher, Marshall should now add $2^4 = 16$. Karen's number must be $672 + 16 = \mathbf{688}$.

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