MATHCOUNTS® Problem of the Week Archive

Vacation Plans – July 15, 2019

Problems & Solutions

The Walton and the Brady families plan to vacation together in an 8-bedroom vacation home. John and Olivia will bring six children — boys, Jason, Ben and Jim-Bob, and girls, Mary Ellen, Erin and Elizabeth. Mike and Carol also will bring six children — boys, Greg, Peter and Bobby, and girls, Marcia, Jan and Cindy. John and Olivia will sleep in one of the bedrooms, and Mike and Carol will stay in another bedroom. Each of the remaining six bedrooms each will be occupied by a pair of children, with six boys using three bedrooms and six girls using three bedrooms.

To assign roommates, Mr. Brady has written the numbers 1, 2, 3 and the letters A, B, C on six slips of paper. After the slips are folded and placed in a bowl, each of the boys will randomly select a folded paper from the bowl, without replacement, and the roommate pairings will be 1 with A, 2 with B and 3 with C. All six slips, then, will be refolded and returned to the bowl so that this process can be repeated for the girls.

Using this method, how many distinct sets of roommate pairings are possible for the boys?

*Each distinct set of roommate pairings will consist of 3 pairs of boys. There are ${}_6C_2 = 15$ ways of selecting the first roommate pairing. There are ${}_4C_2 = 6$ ways of selecting the second pairing. That leaves just ${}_2C_2 = 1$ option for the last pairing of the set. That means there are $15 \times 6 \times 1 = 90$ sets of roommate pairings. But they are not all distinct because each distinct set of pairings can occur in $3! = 6$ different ways. So, there are $90 \div 6 =$ **15** distinct pairings.*

What is the percent probability that Jim-Bob will ***not*** room with one of his brothers?

Jim-Bob is equally likely to be paired with any of the other five boys. Three of those boys are not his brothers. So, the probability that Jim-Bob will not room with one of his brothers is 3/5 = ***60****%.*

Marcia and Mary Ellen both are 13 years old, and Erin, Jan, Cindy and Elizabeth are 12, 10, 8 and 6 years old, respectively. In what fraction of the distinct roommate pairings for the girls is the sum of their ages is 23? Express your answer as a common fraction.

There are ${}_6C_2 = 15$ distinct roommate pairings for the girls. The pairing of Marcia with Jan and the pairing of Mary Ellen and Jan are the two roommate pairings for which the sum of their ages is 23. Those two pairings represent ***2/15*** *of the distinct roommate pairings for the girls.*

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