

MATHCOUNTS[®] Problem of the Week Archive

Camsie's New Job – June 24, 2019

Problems & Solutions

Camsie is starting a new job where she earns n dollars on her n th day of work. What is the total amount that Camsie will earn for the first ten days of work at her new job?

On each of the first 10 days of work at her new job, Camsie will earn 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 dollars, respectively. That's a total of $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 10(1 + 10)/2 = 5(11) = \mathbf{55}$ dollars.

How many days will Camsie have worked at her new job when her cumulative earnings total 595 dollars?

Camsie's cumulative earnings for working the first n days at her new job equal the sum of the integers from 1 to n . Since $1 + 2 + 3 + \dots + n = (n/2)(n + 1)$, we can solve the equation $(n/2)(n + 1) = 595 \rightarrow n(n + 1) = 1190 \rightarrow n^2 + n - 1190 = 0$. Factoring the quadratic yields $(n - 34)(n + 35) = 0$. So, $n - 34 = 0 \rightarrow n = 34$ and $n + 35 = 0 \rightarrow n = -35$. Therefore, Camsie's cumulative earnings will total 595 dollars when she has worked **34** days.

What is the minimum number of days Camsie must work at her new job for her cumulative earnings to exceed 1000 dollars?

Using the same expression for the sum of the integers from 1 to n , we have the following inequality: $(n/2)(n + 1) > 1000 \rightarrow n(n + 1) > 2000$. Simplifying yields the quadratic expression $n^2 + n - 2000$, which cannot be factored. However, we are essentially looking for two consecutive integers, n and $n + 1$, that have a product greater than 2000. Since $40^2 = 1600$ and $50^2 = 2500$, we know the integers are between 40 and 50. The square of 45 is 2025, so we check 44×45 and 45×46 and get 1980 and 2070, respectively. Therefore, we conclude that for Camsie's cumulative earnings to exceed 1000 dollars, she must work at least **45** days.

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