MATHCOUNTS® Problem of the Week Archive

School's Out for Summer – June 3, 2019

Problems & Solutions

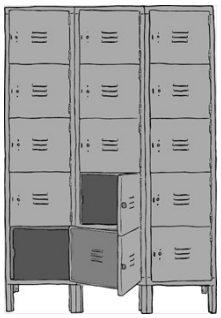
At Walter Whitman Middle School, to celebrate the last day of school, Mrs. Hanks brought in treats for her class. She has chocolate bars, lollipops and gumballs. Mrs. Hanks tells each student they can choose any combination of three treats from what she has. How many different combinations of three treats can one of her students chose?

Let's call the three types of treats A, B and C. There is 1 combination that has one of each type: ABC. There are 3 combinations that each have three of the same type: AAA, BBB and CCC. There are 6 combinations that have two of one type and one of another: AAB, AAC, BBA, BBC, CCA and CCB. These are the only 1 + 3 + 6 = ***10*** *possible combinations of three treats.*

At the end-of-year assembly, WWMS holds a raffle, in which several lucky students will each win a gift certificate to the famous Giorgio's Gelato. There are 10 gift certificates to be raffled to the 6th graders, 15 gift certificates to be raffled to the 7th graders and 20 gift certificates to be raffled to the 8th graders. Each grade has 300 students. Junie is in 8th grade at WWMS. Junie's sister and brother also attend WWMS and are in 7th and 6th grade, respectively. What is the probability that at least one of the three siblings wins a gift certificate? Express your answer as a common fraction.

For each of the gift certificates, each sibling has a 1/300 chance of winning. Junie's brother in 6th grade as a 1/300 chance at each of the 10 gift certificates, so 10/300 = 1/30 chance of winning a gift certificate. Junie's sister in 7th grade, similarly, has a 15/300 = 1/20 chance of winning. Junie has a 20/300 = 1/15 chance of winning. The probability that at least one of them wins a gift certificate is, therefore, 1/30 + 1/20 + 1/15 = ***3/20****.*

Before WWMS students were officially released for summer vacation, the staff had to conduct an official check of all 900 lockers, numbered from 1 to 900. Once the locker check was completed, all 900 lockers remained open. As students exited in an orderly fashion, walking down the hallway single file, the 1st student closed all the lockers. The 2nd student then opened every even numbered locker. The 3rd student changed the status of every locker whose number was a multiple of three – opened it if closed or closed it if opened. The 4th student then did the same for every locker whose number was a multiple of four, and so on through the 900th student who changed the status of locker number 900. When all of the students had left the school, how many lockers were closed?



The lockers that will be open after the 900th student leaves, are all of the lockers whose numbers have an even number of factors. Take 6, for example, its factors are 1, 2, 3 and 6. Student 1 closes the locker, student 2 opens it, student 3 closes it and then student 6 opens it again. A locker number with an odd number of factors will end up closed after the 900th student leaves. Which numbers have an odd number of factors? Only perfect squares have an odd number of factors. The number 1 only has itself as a factor; 4 has 1, 2 and 4 as factors; 9 has 1, 3 and 9 as factors; and so on. This is because factors come in pairs and only perfect squares have 1 pair consisting of the same integer repeated. For example, the factor pairs of 9 are 1 × 9 and 3 × 3. The last perfect square locker number is $30^2 = 900$*, so since there are 30 perfect square locker numbers,* 1^2 *to* 30^2*, after all the students left the school, there were* ***30*** *closed lockers.*

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