MATHCOUNTS® Problem of the Week Archive

2017 National Competition – May 22, 2017

Problems & Solutions

Last week the National Competition concluded, and Luke Robitaille from Texas became the 2017 MATHCOUNTS National Champion. Let's look at some of the problems he had to solve on the way to the top!

What is the maximum possible absolute difference between a two-digit integer and the two-digit integer resulting when the digits are reversed?

We want the largest possible two-digit integer and the smallest possible two-digit integer that have their digits reversed. These are 91 and 19. The difference is 91 – 19 = ***72****.*

How many perfect squares are divisors of the product 1! · 2! · 3! · 4! · 5! · 6! · 7!?

Let's write this product of factorials out in its prime factorization: $2^{16} \times 3^7 \times 5^3 \times 7^1$*. Since we want perfect square factors, we are looking for pairs of primes. The highest perfect square factor is* $2^{16} \times 3^6 \times 5^2 = (2^8 \times 3^3 \times 5^1)^2$*. Since this is the highest factor, we can find the total number of factors by finding how many ways there are to choose pairs of 2s, 3s and 5s. There are 8 pairs of 2s, so 9 ways to pick the number of pairs, from 0 to 8. There are 3 pairs of 3s, so 4 ways to pick the number of pairs, from 0 to 3. Lastly, there is only 1 pair of 5s, so there is 2 ways to pick the number of pairs, 0 or 1. In total, that gives us 9 × 4 × 2 =* ***72*** *perfect squares.*

A college dorm houses math majors and chemistry majors. There is at least one student in each major and no students are majoring in both. Students live in single or double rooms. If four-fifths of the math majors are roommates with six-sevenths of the chemistry majors, what is the least possible number of students living in the dorm?

If 4/5 × M = 6/7 × C, then C/M = 28/30 = 14/15. The least number of students living in the dorm is therefore 14 + 15 = ***29*** *students.*

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