Problems

Last week the national competition concluded, and Daniel Mai from Massachusetts earned the title of MATHCOUNTS National Champion. Let’s look at some of the problems he had to solve on the way to the top!

Sprint #14

Two opposite vertices of a certain square are located at (1, 6) and (−3, 1). If the line $y = mx$ divides this square into two regions of equal area, what is the value of $m$? Express your answer as a common fraction.

Sprint #20

A certain digital lock has a keypad with five buttons labeled 1 to 5. To activate the locking mechanism, a secret code is set with these restrictions: the code must contain three parts, each part will consist of either pressing one button or pressing two buttons simultaneously, and no button may be pressed more than once. For example [3][4][1], [1][2 & 4][5] and [1 & 2][4][3 & 5] are three possible codes. Also, note that [1][2 & 4][5] and [1][4 & 2][5] are indistinguishable. How many distinct secret codes can be set for this lock?

Sprint #29

How many of the first 100,000 positive integers have no single-digit prime factors?