

MATHCOUNTS[®] Problem of the Week Archive

Final Countdown – May 14, 2018

Problems & Solutions

On Sunday, May 13th, 224 middle-school math students participated in the written rounds of the 2018 Raytheon MATHCOUNTS National Competition. On Monday, May 14th, the top 12 competitors will go head to head in the National Countdown Round to determine the 2018 MATHCOUNTS National Champion. Let's solve a few problems from the 2017 National Countdown Round.

In Zagland, people measure length in two units: zigs and zags. A length of x zigs is equal to a length of $(x + 3)$ zags, and a length of $(3x + 7)$ zigs is equal to a length of $(3x + 22)$ zags. What is the value of x ? Express your answer as a common fraction.

We can set up the proportion $x/(x + 3) = (3x + 7)/(3x + 22)$. Cross-multiplying, we get $x(3x + 22) = (x + 3)(3x + 7) \rightarrow 3x^2 + 22x = 3x^2 + 16x + 21 \rightarrow 6x = 21 \rightarrow x = 21/6 = 7/2$.

How many different amounts can be obtained by using one or more coins in a collection of one quarter, one dime, two nickels and four pennies?

*Looking at just the nickels, dimes and quarters, we can see that the following amounts, in cents, can be made 5, 10, 15, 20, 25, 30, 35, 40 and 45. Then using 1, 2, 3 or 4 pennies, the following amounts, in cents, can be made 1 to 4, 6 to 9, 11 to 14, 16 to 19, 21 to 24, 26 to 29, 31 to 34, 36 to 39, 41 to 44, 46 to 49. So, any amount from 1 cent to 49 cents can be made for a total of **49** amounts.*

In a barn, 100 chicks sit peacefully in a circle. Suddenly, each chick randomly pecks the chick immediately to its left or right. What is the expected number of un-pecked chicks?

*If we look at any one particular chick, there is a 1 in 2 chance that it will not be pecked by the chick to its left and a 1 in 2 chance it will not be pecked by the chick to its right. This means there is a 1 in 4 chance it will not be pecked at all. This probability is the same for every chick in the circle. Since there are 100 chicks total, we can expect 1 in 4 to be un-pecked or **25** chicks.*

Alternatively, for any one chick, there are four possible outcomes. Being pecked only by the chick to its left, being pecked only by the chick to its right, being pecked by both the chicks or being pecked by neither of the chicks. Each of the four outcomes is equally probable, so there is a 1 in 4 chance of any chick being un-pecked. Since there are 100 chicks in the circle we can expect, $1/4 \times 100 = 25$ un-pecked.

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