MATHCOUNTS® Problem of the Week Archive

Final Countdown – May 13, 2019

Problems & Solutions

On Sunday, May 12th, 224 middle-school math students participated in the written rounds of the 2019 Raytheon MATHCOUNTS National Competition. On Monday, May 13th, the top 12 competitors will go head to head in the National Countdown Round to determine the 2019 MATHCOUNTS National Champion. Let's solve a few problems from the 2018 National Countdown Round.

National Countdown #21

In square units, what is the area of the triangle with vertices P(−2, 1), Q(3, 8) and R(9, 3)? Express your answer as a decimal to the nearest tenth.

One way to solve this problem is by graphing the triangle with vertices P(−2, 1), Q(3, 8) and R(9, 3) on a coordinate grid. Then, we construct a rectangle with vertices P(−2, 1), O(−2, 8), N(9, 8) and M(9, 3) that circumscribes this triangle. We see that rectangle MNOP is composed of triangle PQR and right triangles POQ, RNQ and PMR, and that the area of triangle PQR is the area rectangle MNOP minus the areas of right triangles POQ, RNQ and PMR. Therefore, the area of triangle PQR is (7)(11) − (1/2)(5)(7) − (1/2)(5)(6) − (1/2)(2)(11) = 77 − 35/2 − 15 − 11 = 51 − 35/2 = (102 − 51)/2 = 67/2 = ***33.5*** *units*^{2}*.*
Another way to solve this problem is by using the shoelace method, which says that the are of a closed figure with vertices $A(x_1, y_1)$*,* $B(x_2, y_2)$ *and* $C(x_3, y_3)$ *is* $|(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)|/2$*. So, for the area of triangle PQR, we have |((−2)(8) + (3)(3) + (9)(1)) − ((1)(3) + (8)(9) + (3)(−2))|/2 = |(−16 + 9 + 9) − (3 + 72 − 6)|/2 = |2 − 69|/2 = 67/2 =* ***33.5*** *units*^{2}*.*

National Countdown #16

What is the greatest prime factor of $2^{13} + 2^{11} - 10$?

The expression $2^{13} + 2^{11} - 10$ *can be rewritten* $(2^{11})(4 + 1) - 10 = (2^{10})(10) - 10 = (10)(2^{10} - 1) = (10)((2^5)^2 - 1) = (10)(2^5 + 1)(2^5 - 1) = 10(33)(31) = 2 \times 5 \times 3 \times 11 \times 31$*. Thus, the greatest prime factor of this expression is* ***31****.*

National Countdown #13

The four largest numbers in a set of seven numbers have a mean of 10. The four smallest numbers in the same set have a mean of 5. What is the least possible sum of the seven numbers?

Let a, b, c, d, e, f and g represent the seven numbers in ascending order. We are told that (d + e + f + g)/4 = 10, so d + e + f + g = 40. We also are told that (a + b + c + d)/4 = 20, so a + b + c + d = 80. That means a + b + c + d + d + e + f + g = 40 + 20, and a + b + c + d + e + f + g = 60 − d. In order to minimize the sum of these seven numbers, which is 60 − d, we need to maximize the value of d. The greatest possible value of d occurs when d = e = f = g = 10. Therefore, the least possible value of the sum of these seven numbers is 60 − d = 60 − 10 = ***50****.*

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