

# MATHCOUNTS® Problem of the Week Archive

## National Competition – May 8, 2017

### Problems & Solutions

On Sunday, May 14th, 224 of the nation's smartest middle-school math minds will be in Orlando, FL for the 2017 Raytheon MATHCOUNTS National Competition. The stakes are high, and the problems will be tough. Here are a few sprint round problems national competitors solved in 2016.

Ms. Parker has three types of inhabitants on her farm: humans, horses and spiders. The humans and horses are both mammals, but humans have 2 legs each and while each horse has 4 legs. The spiders each have 8 legs and are not mammals. If the number of human legs, horse legs and spider legs are all equal, what is the ratio of mammal inhabitants to the total number of inhabitants? Express your answer as a common fraction.

Let's use the variable  $M$  for the number of humans,  $R$  for the number of horses and  $S$  for the number of spiders. If the number of legs of each animal is the same, then we know  $2M = 4R = 8S$ . We can put each variable in terms of  $S$ . We get  $2M = 8S$  or  $M = 4S$  and  $4R = 8S$  or  $R = 2S$ . The ratio of the number of mammals to total number of inhabitants is  $(M + R)/(M + R + S) = (4S + 2S)/(4S + 2S + S) = (4 + 2)/(4 + 2 + 1) = \mathbf{6/7}$ .

Olar runs one lap around Carpenter Park in 6 minutes. Nori runs one lap around Carpenter Park in 8 minutes. If Olar and Nori start together and run in the same direction around the park, how many laps will Olar have completed the first time he and Nori are together again at the starting point?

Since Olar runs a lap every 6 minutes and Nori runs a lap every 8 minutes, they will be back together again at the starting point for the first time at 24 minutes – the LCM of 6 and 8. In 24 minutes Olar will have run  $24 \div 6 = \mathbf{4}$  laps.

For what digit  $A$  do the 3-digit numbers  $A43$  and  $A79$  have a product of  $43A,597$ ?

If you set up the multiplication of  $A43$  and  $A79$ , then the  $43$  at the beginning of the product is  $A^2$  plus some single digit carried over. The only way  $43$  can be obtained from a square number plus a single digit number is if the square number is  $36$  and the single digit is  $7$ . This makes  $A = \mathbf{6}$ .

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### ***Problems***

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