MATHCOUNTS® Problem of the Week Archive

Countdown to Nationals – May 1, 2017

Problems & Solutions

We are officially two weeks away from this year's MATHCOUNTS National Competition Countdown Round! The top 12 competitors in the country will go head to head to answer questions in 45 seconds or less. Let's look at some questions they answered last year. How fast can you solve them?

To make a compact disc, a machine creates a circular disc with diameter 120 mm and then punches a center hole with diameter 15 mm. When the hole is punched, what fraction of the original area is removed? Express your answer as a common fraction.

*The fraction of the area removed will be the ratio of the squares of the diameters of the center hole and the circular disc or $15^2/120^2 = (15/120)^2 = (1/8)^2 =$ **1/64**.*

Todd tells Juanita that he is thinking of a 3-digit positive integer. The integer has 12 positive factors. The sum of two of its factors is 23 and the difference of those two factors is 1. What number is Todd thinking of?

*The sum of two of the factors is 23 and they differ by one, so these two factors must be 12 and 11. The product is $12 \times 11 = 132$. Let's check to make sure this number has 12 positive factors. The prime factorization of 132 is $2^2 \times 3 \times 11$. There are 3 ways to choose the number of 2s – 0, 1 or 2; 2 ways to choose the number of 3s – 0 or 1; and 2 ways to choose the number of 11s – 0 or 1. That means the number of factors is $3 \times 2 \times 2 = 12$. This satisfies the conditions, so **132** is the number Todd is thinking of.*

The least positive integer that is divisible by 2, 3, 4 and 5, and is also a perfect square, perfect cube, 4th power and 5th power, can be written in the form a^b for positive integers a and b. What is the least possible value of $a + b$?

*To guarantee a number is a perfect square, cube, 4^{th} or 5^{th} power, we need to raise the number to the power of $3 \times 4 \times 5 = 60$. We also want the number to be divisible by 2, 3, 4 and 5. If we raise $2 \times 3 \times 5 = 30$ to any power, 2 or greater, then it will be divisible by these integers. So, 30^{60} would satisfy our conditions. The least possible value of a + b is therefore $30 + 60 =$ **90**.*

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