

# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

## Counting Down to Nationals – April 29, 2019

### Problems & Solutions

The countdown round is often the most anticipated and exciting part of competition for Mathletes<sup>®</sup> and spectators alike. Here are some of our favorite countdown round problems from the 2019 competitions.

#### School #16

On a certain farm, each chicken has two feet and each rabbit has four feet. If the combined number of chickens and rabbits on the farm is 100 and there are a total of 260 feet on these animals, how many chickens are there?

*Let  $c$  and  $r$  represent the numbers of chickens and rabbits, respectively, on this farm. We can write the following equations:  $2c + 4r = 260$  and  $c + r = 100$ . Dividing both sides of the first equation by 2 yields  $c + 2r = 130$ . If we subtract the two equations  $c + 2r = 130$  and  $c + r = 100$ , we get  $(c + 2r) - (c + r) = 130 - 100 \rightarrow r = 30$ . So, there are 30 rabbits on this farm and  $100 - 30 = 70$  chickens.*

#### Chapter #59

Seven consecutive even integers have a sum of 406. What is the sum of the least and the greatest of the seven integers?

*For a list of consecutive integers, the mean is equal to the median of the integers. So, for these seven consecutive integers, the mean and median are both equal to  $406/7 = 58$ . Since there are seven integers in this list, the median is middle integer, which is also the fourth integer, when the list is arranged ordered. If the fourth integer is 58, then the least of the seven integers is three less than 58, or 55; the greatest of these integers is three more than 58, or 61. The sum of the least and greatest of the seven integers, then, is  $55 + 61 = 116$ .*

#### State #43

Scott is thinking of a positive integer that is one more than, or one less than a multiple of 4. Quincy is thinking of an integer that is one more than, or one less than a multiple of 10. Stella is thinking of an integer with an even number of positive integer divisors. If all three are thinking of the same integer  $n$ , what is the least possible value of  $n$ ?

*If Stella's number has an even number of positive divisors, it must not be a perfect square. Scott's number is positive and is one more or one less than a multiple of 4. Listing some multiples of 4, we have 0, 4, 8, 12, 16, 20, 24, .... So, excluding perfect squares, Scott's number could be, 3, 5, 7, 11, 13, 15, 17, 19, 21, 23, .... Quincy's number, which must also be positive, is one more or one less than a multiple of 10. Let's see if any numbers on the list we've started of possible numbers Scott is thinking of is one more or one less than a multiple of 10. We see that the list we've started includes 11 and 21, both of which are one more than a multiple of 10, and 19, which is one less than a multiple of 10. So, the least possible value of the integer that these three are thinking of is  $n = 11$ .*

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