MATHCOUNTS® Problem of the Week Archive

Let the Games Begin! – April 23, 2018

Problems & Solutions

This season, each team in the Davis City softball league will play each other team exactly once. If a total of 15 games will be played, how many teams are in the league?

Let n represent the number of teams in the softball league. Each team plays every other team once, for a total of 15 games, so we have $n(n-1)/2 = 15 \rightarrow n^2 - n = 30 \rightarrow n^2 - n - 30 = 0$. *Factoring this quadratic equation, we get* $(n-6)(n+5) = 0 \rightarrow n = 6$ *or* $n = -5$. *The number of teams must be positive, so it follows that there are* ***6*** *teams in the league.*

The mayor's two daughters each play for a different team in the Davis City softball league. This season, the mayor's schedule will allow him to attend only one softball game. If he randomly selects which one of the 15 games to attend, what is the probability that the mayor will select a game in which at least one of his daughters is playing? Express your answer as a common fraction.

Since each team plays each other team once, we know that this season each of the mayor's daughters will play in 5 games. That's 5 + 5 = 10 games; but the game in which his daughter's teams play each other has been counted twice. So, out of the 15 games played, at least one of the mayor's daughters will play in 9 games. The probability of the mayor randomly selecting one of those 9 games to attend is 9/15 = ***3/5****.*

Next year, in order to extend the season beyond 15 games, after every team plays each other team once, the league will have every team play each other team a second time. If the order of the 15 games in the first half of the season and the order of the 15 games in the second half of the season are chosen at random, what is the probability that the two teams that play in the 1st game of the season will be the same two teams that play in the 30th game of the season? Express your answer as a common fraction.

The15 games played in the second half of the season are not dependent on the 15 games played in the first half of the season. So we need only consider the number of possible match-ups for the last game of the season. That's ${}_6C_2 = 6!/(2!4!) = (6 \times 5)/2 = 15$ *different match-ups. One of these match-ups will be the same as the match-up for the 1st game. Therefore, the probability that the same two teams that played in the 1st game will also play in the 30th game is* ***1/15****.*

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