

# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

## Follow the Rules – April 22, 2019

### Problems & Solutions

Define  $x @ y$  as  $(x^3 - y)/x$ , for distinct positive integers  $x$  and  $y$ . What is the value of  $5 @ 10$ ?

Evaluating  $5 @ 10$  yields  $(5^3 - 10)/5 = (125 - 10)/5 = 115/5 = 23$ .

Define  $a \# b$  as  $a^2 - b^2 - ab$ , for real numbers  $a$  and  $b$ . What is the value of  $5 \# (4 \# 3)$ ?

Let's first evaluate the expression inside the parentheses. We have  $4 \# 3 = 4^2 - 3^2 - 4(3) = 16 - 9 - 12 = -5$ . We now evaluate  $5 \# (-5)$  to get  $5^2 - (-5)^2 - 5(-5) = 25 - 25 + 25 = 25$ .

Define  $m \$ n$  as  $m^2 + 2m/n$  and define  $m \& n$  as  $(m^2 - n^2 + mn)/(2n)$ . What is the value of  $(4 \$ 2) \& 10$ ?

Again, let's first evaluate the expression inside the parentheses. We have  $(4 \$ 2) = 4^2 + 2(4)/2 = 16 + 8/2 = 16 + 4 = 20$ . We now evaluate  $20 \& 10$  to get  $[20^2 - 10^2 + 20(10)]/[2(10)] = (400 - 100 + 200)/20 = 500/20 = 25$ .

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### ***Problems***

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Define  $m \$ n$  as  $m^2 + 2m/n$  and define  $m \& n$  as  $(m^2 - n^2 + mn)/(2n)$ . What is the value of  $(4 \$ 2) \& 10$ ?