MATHCOUNTS® Problem of the Week Archive

State Countdown – April 16, 2018

Problems & Solutions

Here are a few of the problems that competitors had to solve as they faced off in head-to-head matches during the 2018 State Countdown Round.

The first five terms of an arithmetic sequence are x, y, $x + 6$, z and 37, in that order, for some numbers x, y and z. What is the value of y?

[Countdown #4]

*Since we are told that the five terms form an arithmetic sequence, we know that the difference, d, between consecutive terms is constant. The value of the third term is two of these constant differences away from the value of the first term, so we have $x + 2d = x + 6 \rightarrow 2d = 6 \rightarrow d = 3$. We now know that the difference between consecutive terms is 3. Similarly, since the value of the fifth term is 4 constant differences away from the value of the first term, we have $x + 4d = 37 \rightarrow x + 4(3) = 37 \rightarrow x + 12 = 37 \rightarrow x = 25$. Finally, since $y = x + d$, it follows that $y = 25 + 3 =$ **28**.*

Mr. Thomas wants to divide all seven students in his algebra class into three teams of at least two students each. How many different ways are there for him to divide his students into teams?

[Countdown #16]

*Based on the information provided, Mr. Thomas will divide the seven students into three teams, two with 2 students and one with 3 students. There are $_7C_3 = (7!)/(4! \times 3!) = 35$ ways to select the students for the team of 3 students. There are $_4C_2 = (4!)/(2! \times 2!) = 6$ ways to split the remaining four students into two teams of 2 students each. However, this double counts each grouping, so there are only $6/2 = 3$ distinct ways to split four students into two teams of 2 students each. That's a total of $35 \times 3 =$ **105** ways for Mr. Thomas to divide his students into teams as described.*

Yvonne takes a 10-question true and false test. If she randomly guesses on every question, what is the probability she gets at least 75% of the questions correct? Express your answer as a common fraction.

[Countdown #33]

*Since 75% of 10 is 7.5, Yvonne can answer at most 2 questions incorrectly to get at least 75% of them correct. There are $2^{10} = 1024$ ways for Yvonne to answer the 10 questions on a true and false test. There is $\underline{1}$ way to answer zero questions incorrectly. There are $\underline{10}$ ways to answer one question incorrectly. There are $_{10}C_2 = (10!)/(8! \times 2!) = \underline{45}$ ways to answer two questions incorrectly. That's a total of $1 + 10 + 45 = 56$ ways for her to answer at least 75% of the questions correctly, for a probability of $56/1024 =$ **7/128**.*

Trapezoid ABCD has parallel bases AB and DC. The area of triangle ABD is 6 units^2, and the area of triangle ACD is 9 units^2. What is the area of the trapezoid, in square units?

[Countdown #67]

*Triangle ABD, with height h and base length AB, has area 6 units^2. So, $(1/2) \times AB \times h = 6 \rightarrow AB = 12/h$. Triangle ACD, with height h and base length DC, has area 9 units^2. So, $(1/2) \times DC \times h = 9 \rightarrow DC = 18/h$. Trapezoid ABCD, then, has area $(1/2) \times (12/h + 18/h) \times h \rightarrow (1/2) \times (30/h) \times h = (1/2) \times 30 =$ **15** units^2.*

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