MATHCOUNTS® Problem of the Week Archive

Congrats, State Winners! – April 9, 2018

Problems & Solutions

Congratulations to everyone who competed in the 2018 State Competition, especially to the winners who are headed to Washington, DC next month to compete in the 2018 Raytheon MATHCOUNTS National Competition! Now that the competitions have officially wrapped up, let's take a look at a few of the challenging problems from the 2018 State Competition.

The orbital period of a planet is the time it takes to make one revolution around its sun. In a distant solar system, the giant planet Flion makes 5 orbits of its sun in the time it takes its planetary neighbor Reflurn to make 2 orbits. Flion makes 19 orbits of its sun in the time it takes the comet Hathov to make 3 orbits. What is the ratio of Hathov's orbital period to Reflurn's orbital period? Express your answer as a common fraction.

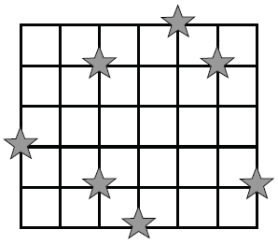
[Sprint #16]

Let T represent the time of one orbit, called the orbital period, and apply a subscript F when applied to Flion, H when applied to Hathov, and R when applied to Reflurn. Then, we want T_H/T_R, so let's express T_H in terms of T_F and T_F in terms of T_R, since we are given the relationship between Flion and each of Hathov and Refurn. We have $5T_F = 2T_R \rightarrow T_F = (2/5)T_R$, and $19T_F = 3T_H \rightarrow T_H = (19/3)T_F = (19/3)(2/5)T_R = (38/15)T_R$. So, $T_H/T_R =$ ***38/15****.*

It costs a geometrist $400 to set up machinery and $3 to produce each bigon. If she makes the bigons to order and sells the bigons for $8 each, what is the least positive number of bigons she has to sell to avoid losing money?

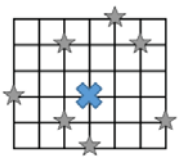
[Target #3]

There is a gain of $8 − $3 = $5 for each bigon sold. With $400 upfront costs to make up for at $5 per bigon, to break even, she must sell $400/$5 = ***80*** *bigons.*



The grid shows the streets in downtown Geometrika. All streets run either north-south or east-west. Adjacent parallel streets are 1 km apart. The seven restaurants in Geometrika are located at the street corners labeled with stars. To get between any two locations the city, Henry must walk on the streets. Ignoring street widths, he positions himself on the streets such that he minimizes the average of the walking distances between himself and the restaurants in the city. From Henry's chosen position, what is the average of the walking distances between him and each restaurant? Express your answer as a common fraction.

[Team #4]



When one wishes to minimize the sum of distances in along one dimension, the optimum position is at the median position. Here we have two dimensions, with each working independently of the other. There are 7 restaurants, so the fourth acts as the median. The blue cross marks the optimum position—the same horizontal coordinate as the fourth restaurant from left to right and the same vertical coordinate as the fourth restaurant from top to bottom. The cumulative horizontal distances, as a number of kilometers, is 3 + 1 + 1 + 0 + 1 + 2 + 3 = 11; the cumulative vertical distances as a number of kilometers is 3 + 2 + 2 + 0 + 1 + 1 + 2 = 11. The total distance of travel to go separately from the optimum point to each restaurant, is the sum of these two values, 11 km + 11 km = 22 km. The average distance to each restaurant is this sum divided by 7, thus ***22/7*** *km.*

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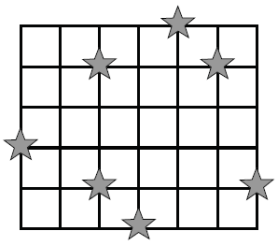

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