

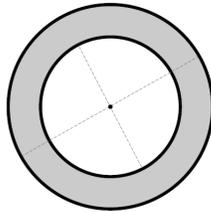
MATHCOUNTS[®] Problem of the Week Archive

The Road to Nationals – April 8, 2019

Problems & Solutions

“Job well done!” to all the Mathletes[®] who participated in the MATHCOUNTS Competition Series at the school, chapter and state levels. And congratulations to the 224 students who are headed to Orlando next month to compete in the 2019 Raytheon MATHCOUNTS National Competition! Let’s look at some of the challenging problems competitors faced on the road to Nationals!

School Target #8



The two concentric circles shown have diameters a mm and b mm, where a and b are integers with $a < b$. The gray region between the two circles has area 48π mm². What is the sum of all possible values of b ?

The circle of diameter b mm has radius $b/2$ mm and area $\pi(b/2)^2 = b^2\pi/4$ mm². The circle of diameter a mm has radius $a/2$ mm and area $\pi(a/2)^2 = a^2\pi/4$ mm². Since $a < b$, it follows that the gray region between the outer circle of diameter b mm and the inner circle of diameter a mm has area, in square millimeters, $b^2\pi/4 - a^2\pi/4 = (\pi/4)(b^2 - a^2)$, which we are told equals 48π mm². So, $(\pi/4)(b^2 - a^2) = 48\pi$ and $b^2 - a^2 = 48\pi(4/\pi) = 192$, and $b^2 - a^2 = 192$ can be rewritten as $(b - a)(b + a) = 192$. We know that $192 = 1 \times 192 = \underline{2} \times \underline{96} = 3 \times 64 = \underline{4} \times \underline{48} = \underline{6} \times \underline{32} = \underline{8} \times \underline{24} = \underline{12} \times \underline{16}$. The five underlined factor pairs can be written as the sum and difference of two integers as follows:

$$\begin{aligned} 2 \times 96 &= (49 - 47)(49 + 47) \rightarrow a = 47 \text{ and } b = \underline{49} \\ 4 \times 48 &= (26 - 22)(26 + 22) \rightarrow a = 22 \text{ and } b = \underline{26} \\ 6 \times 32 &= (19 - 13)(19 + 13) \rightarrow a = 13 \text{ and } b = \underline{19} \\ 8 \times 24 &= (16 - 8)(16 + 8) \rightarrow a = 8 \text{ and } b = \underline{16} \\ 12 \times 16 &= (14 - 2)(14 + 2) \rightarrow a = 2 \text{ and } b = \underline{14} \end{aligned}$$

The five possible values of b are 49, 26, 19, 16 and 14 and their sum is $49 + 26 + 19 + 16 + 14 = \mathbf{124}$.

Chapter Sprint #23

If A represents a digit such that the sum of the two-digit numbers $2A$, $3A$ and $4A$ is the three-digit number $10A$, what is the value of A ?

The sum of the three values is $(20 + A) + (30 + A) + (40 + A) = 90 + 3A$, but we are told that their sum is $100 + A$. So, $90 + 3A = 100 + A \rightarrow 2A = 10 \rightarrow A = \mathbf{5}$.

State Team #5

Mr. Schwin has a large jar containing M&Ms, each with the letter "m" stamped on it. He removes 1000 candies from the jar and removes the letter "m" from each one. He then returns all of the M&Ms to the jar. After thoroughly mixing up the candies in the jar, he randomly removes 1000 candies from the jar and finds that 245 of them do not contain the letter "m". What is the expected number of M&Ms in the jar? Express your answer to the nearest whole number.

When 1000 candies are randomly drawn from the jar and 245 of them do not have the "m," that means our best estimator of the probability of a candy not having the "m" is $245/1000 = 0.245$. Thus, if there are n candies in total, then $0.245n$ candies should not have the "m." Now, we know the number of candies that are missing the "m" is actually 1000, so $0.245n \approx 1000$. Thus, $n \approx 1000/0.245 \approx \mathbf{4082}$ is the expected number of M&Ms in the jar.

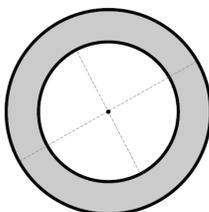
MATHCOUNTS[®] Problem of the Week Archive

The Road to Nationals – April 8, 2019

Problems

“Job well done!” to all the Mathletes[®] who participated in the MATHCOUNTS Competition Series at the school, chapter and state levels. And congratulations to the 224 students who are headed to Orlando next month to compete in the 2019 Raytheon MATHCOUNTS National Competition! Let’s look at some of the challenging problems competitors faced on the road to Nationals!

School Target #8



The two concentric circles shown have diameters a mm and b mm, where a and b are integers with $a < b$. The gray region between the two circles has area 48π mm². What is the sum of all possible values of b ?

Chapter Sprint #23

If A represents a digit such that the sum of the two-digit numbers $2A$, $3A$ and $4A$ is the three-digit number $10A$, what is the value of A ?

State Team #5

Mr. Schwin has a large jar containing M&Ms, each with the letter “m” stamped on it. He removes 1000 candies from the jar and removes the letter “m” from each one. He then returns all of the M&Ms to the jar. After thoroughly mixing up the candies in the jar, he randomly removes 1000 candies from the jar and finds that 245 of them do not contain the letter “m”. What is the expected number of M&Ms in the jar? Express your answer to the nearest whole number.