

# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

## Combining Shapes – April 1, 2019

### Problems & Solutions

The radius of circle N is 3 inches, and the radius of circle O is 4 inches. What is the radius of circle P whose area is the sum of the areas of circles N and O?

The formula for the area of a circle is  $A = \pi r^2$ . So, the area of circle N is  $3^2\pi = 9\pi \text{ in}^2$ , and the area of circle O is  $4^2\pi = 16\pi \text{ in}^2$ . The area of circle P is  $9\pi + 16\pi = 25\pi \text{ in}^2$ . So, for circle P, we have  $\pi r^2 = 25\pi \rightarrow r^2 = 25 \rightarrow r = \mathbf{5}$  inches.

A small square has side length 7 cm, and a medium square has side length 24 cm. What is the side length of a large square whose area is the sum of the areas of the small and medium squares?

The formula for the area of a square is  $A = s^2$ . So, the area of the small square is  $7^2 = 49 \text{ cm}^2$ , and the area of the medium square is  $24^2 = 576 \text{ cm}^2$ . The area of the large square is  $49 + 576 = \text{cm}^2$ . So, for the large square, we have  $s^2 = 625$  and  $s = \mathbf{25}$  cm.

The side length of a small equilateral triangle is 10 feet, and the side length of a medium equilateral triangle is 24 feet. What is the side length of a large equilateral triangle whose area is the sum of the areas of the small and medium equilateral triangles?

The formula for the area of an equilateral triangle is  $A = s^2\sqrt{3}/4$ . The area of the small equilateral triangle is  $10^2\sqrt{3}/4 = 100\sqrt{3}/4 = 25\sqrt{3} \text{ ft}^2$ , and the area of the medium equilateral triangle is  $24^2\sqrt{3}/4 = 576\sqrt{3}/4 = 144\sqrt{3} \text{ ft}^2$ . The area of the large equilateral triangle is  $25\sqrt{3} + 144\sqrt{3} = 169\sqrt{3} \text{ ft}^2$ . So, for the large equilateral triangle, we have  $s^2\sqrt{3}/4 = 169\sqrt{3} \rightarrow s^2 = 169 \times 4 \rightarrow s = \sqrt{(169 \times 4)} = \sqrt{169 \times 4} = 13 \times 2 = \mathbf{26}$  cm.

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### ***Problems***

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