MATHCOUNTS® Problem of the Week Archive

A Look Back at Chapter – March 26, 2018

Problems & Solutions

This week, the 2018 State Competitions will wrap up, and 224 state competitors will begin preparing to compete at the Raytheon MATHCOUNTS National Competition. Let's take a look back at a few of the chapter problems they encountered on their way to state.

Dak has a quarter, a dime, a nickel and a penny. How many different amounts can be obtained by using one or more of the coins in Dak's collection?

[Sprint #16]

The values of the 25¢, 10¢, 5¢ and 1¢ are separated enough that duplicated sums cannot occur. In any possible mix of the coins, the quarter can be included or excluded (2 options), likewise for the dime (2 options), the nickel (2 options), and the penny (2 options). Each of these 4 inclusion-exclusion cases is independent of the others, so there are 2 × 2 × 2 × 2 = 16 possible mixes of coins; however, one of these, excluding all 4 coins, does not meet the criterion that at least one coin must be used. Therefore, there are ***15*** *possible amounts.*

Aiden and Bryce are racing around a race track. They begin together at the starting line, and Aiden's car completes a lap every 44 seconds, while Bryce's car completes a lap every 40 seconds. How many seconds after they begin the race will Aiden and Bryce first reach the starting line at the same time?

[Target #5]

Let a and b be the number of laps that Aiden and Bryce, respectively, make when they meet again at the starting line. Because the requisite meeting place is at the starting line, that involves a whole number of laps for each, thus making both a and b to be integers. The times involved are 44a seconds and 40b seconds, which must be equal to be a meeting. Now, 44a seconds = 40b seconds, and manipulating algebraically yields b/a = 44/40 =11/10, which cannot be reduced further. Therefore, Bryce takes 11 laps while Aiden takes 10 laps, corresponding to 10 × 44 seconds = 11 × 40 seconds = ***440*** *seconds.*

Noah is combining gummy bears and jelly beans in equal parts to create a mixture that he will sell. The bears cost him $20 for 8 pounds, while the beans cost $14 for 4 pounds. He wants his cost to be 40% of his selling price. At what price per pound should he sell the mixture?

[Team #9]

*To make one pound of equal mix, there needs to be one-half pound gummy bears and one-half pound of jelly beans. Gummy bears cost $20 for 8 pounds, so $20/(8 lb) = $2.50/lb, and one-half pound would cost $1.25. Jelly beans cost $14 for four pounds, so $14/(4 lb) = $3.50/lb, and one-half pound would cost $1.75. The total cost to make one pound of the mix is $1.25 + $1.75 = $3.00, which is to be 2/5 (that is, 40 % = 40/100) of the selling price, so the selling price needs to be $3.00 × 5/2 = $****7.50****.*

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