MATHCOUNTS® Problem of the Week Archive

The Pythagorean Triple Threat – March 25, 2019

Problems & Solutions

The Pythagorean Triple Threat is a coed softball team. League rules require the batting order to alternate boy-girl-boy-girl… (or girl-boy-girl-boy…). If there are five girls and five boys on the team, how many different batting orders are possible?

Let's start by looking at the boys and girls separately. There are 5! = 120 orders in which the girls can bat and there are 5! = 120 orders in which the boys can bat. Each of the girls orders can pair with each of the boys orders, which means there are (120)(120) = 14,400 "pairings" of the boys and girls orders. Additionally, if try to use a "pairing of orders" to create a batting order, we see that the boys can start OR the girls can start which means there are actually 2(14,400) = ***28,800*** *possible batting orders.*

Eight boys and seven girls showed up to one of the team's games. League rules limit each team to 10 players at a time (five girls and five boys) per game. Given this rule, how many combinations of 10 players are there among the 15 boys and girls who showed up at this particular game?

Let's look at the eight boys first. Since only five boys can play, we calculate "8 choose 5" and get $_8C_5 = 8!/(5!3!) = 56$*. So, there are 56 possible combinations of boys. Now let's look at the seven girls. Since only five girls can play, we calculate "7 choose 5" and get* $_7C_5 = 7!/(5!2!) = 21$*. So, there are 21 possible combinations of girls. Each of the 56 combinations of five boys can be combined with each of the 21 combinations of five girls. Thus there are 56(21) =* ***1176*** *possible combinations of players.*

At a different game, four boys and six girls showed up. Even though only four boys showed up, they had a total of ten players, so the team could play, provided they followed batting order rule. In order to keep the required alternating pattern in the batting order, each boy will bat more often than each girl. Representing the six girls with A, E, I, O, U and Y and the four boys with J, K, L and M, the coach decides to following this batting order: A, J, E, K, I, L, O, M, U, J, Y, K, A, L, E, M…. According to this line-up, J is next to bat following A. How many total times will E bat before the next time that J is next to bat following A?

This can be solved by simply listing the order until A is followed by J for a second time. We have A, J, E, K, I, L, O, M, U, J, Y, K, A, L, E, M, I, J, O, K, U, L, Y, M, A, J…. As shown, E bats ***2*** *times before the next time that J is next to bat following A.*

Taking a more logical approach, let's consider a complete batting cycle for the girls to occur when all six girls have batted in order (A, E, I, O, U, Y). Similarly, a complete batting cycle for the boys occurs when all four boys have batted in order (J, K, L, M). Based on the coach's batting order, the next time that J is next to bat following A, is when the boys will be starting a new batting cycle immediately after the girls start a new cycle. Since there are four boys and six girls, we know that when the boys have done two complete batting cycles and the girls have done 1 1/3 complete batting cycles. Since 1/3 of a complete batting cycle for the girls is two girls (6 ÷ 3 = 2), we know that A and E both will have batted a second time, while I, O, U and Y each have batted once. So, once those four girls each have batted a second time, all four boys each will have batted three times. In other words, the girls will have done two complete batting cycles, and the boys will have done three complete batting cycles. That means the J will be next to bat after A bats. Since we know that each girl will have batted twice, it follows that E will have batted ***2*** *times before the next time that J is next to bat following A.*

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