Get Ready for State – March 5, 2018

Problems & Solutions

This month the 2018 MATHCOUNTS State Competitions will take place, so let's look back at some of the 2017 State Sprint round problems and solve them in preparation.

Andre can complete 5/6 of a job in 3/4 of the time that it takes Michael to do the whole job. What is the ratio of the rate at which Andre works to the rate at which Michael works? Express your answer as a common fraction.

[Sprint #10]

The rate in this problem is the number of jobs completed divided by the time to complete them. Let t be the amount of time Michael takes to do one job, so Michael's rate is 1 job/t. Andre's rate is (5/6 job)/(3/4)t = 5/6 × 4/3 job/t = 20/18 job/t = 10/9 job/t, which is **10/9** *times Michael's rate.*

What is the least positive base-10 integer that can be written as a 4-digit number in base 3 and as a 3-digit number in base 4?

[Sprint #15]

The least positive 4-digit value in base 3 is $1000_3 = 3^3 = 27_{10}$. The least positive 3-digit value in base 4 is $100_4 = 4^2 = 16_{10}$. The least value satisfying both is the greater of the two, namely **27**. *Checking, we see that $27_{10} = 1000_3$ and $27_{10} = 123_4$.*

What four-digit number has tens digit 2 and units digit 8, is a multiple of 16, and when its digits are reversed the result is also a multiple of 16?

[Sprint #23]

Let A be the digit for the thousands place and B the digit for the hundreds place. The number we are interested in is AB28. Dividing 28 by 16 yields a remainder of 12 and dividing 100 by 16 yields a remainder of 4. Therefore, 128 plus any multiple of 400 is divisible by 16, which means B must be odd. That yields 25 options, so let's look at the reverse value, 82BA. Dividing 8200 by 16 yields a remainder of 8, so dividing BA by 16 must also yield a remainder of 8, which yields only 6 options for BA: 08, 24, 40, 56, 72, 88. Remember that B must be odd, so only 56 and 72 are candidates. Indeed, **6528** *is divisible by 16, but 2728 is not.*

MATHCOUNTS® Problem of the Week Archive

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