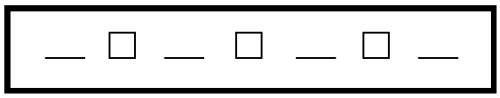
MATHCOUNTS® Problem of the Week Archive

Odds and Evens – March 4, 2019

Problems & Solutions

The sum of any four consecutive odd integers is a multiple of eight. For example, 1 + 3 + 5 + 7 = 16 and 5 + 7 + 9 + 11 = 32. What is the product of the greatest and least of four consecutive odd integers whose sum is 64?

If we look at the sums of four consecutive positive odd integers in order beginning with 1 + 3 + 5 + 7 = 16, then 3 + 5 + 7 + 9 = 24, 5 + 7 + 9 + 11 = 32, 7 + 9 + 11 + 13 = 40, we see that these sums form an arithmetic progression where the common difference is 8. The first seven sums are 16, 24, 32, 40, 48, 56 and 64. Since the seventh sum in this sequence is 64, it is the sum of the four consecutive odd integers beginning with the seventh positive odd integer. The seventh positive odd integer is 13, and 13 + 15 + 17 + 19 = 64. The product of the greatest and least of these four integers is 13 × 19 = ***247****.*



Using the figure, unique mathematical expressions are created by replacing each blank with a different number from 2, 4, 6 and 8 and by replacing each box with a different operator from +, − and ÷. How many of these expressions, when evaluated, have an integer result?

For the expression to have an integer result, the expression must contain one of the four quotients: 8 ÷ 4, 8 ÷ 2, 6 ÷ 2, 4 ÷ 2. For each of these quotients there are 3 possible locations since the quotient can appear at the beginning, middle or end of the expression. This gives us 4 × 3 = 12 options right now. For each of these 12 options, there are 2 ways the other two numbers can be arranged and 2 ways the other two operators can be arranged. Therefore, for each of the four quotients, the remaining numbers and operators can be arranged in 2 × 2 = 4 different ways. Thus, the number of unique expressions that result in an integer when evaluated is 12 × 4 = ***48****.*

What is the positive difference between the greatest and least possible integer results of the expressions in the previous problem?

If the expression contains the quotient 8 ÷ 4, the possible results are −2 and 6. If the expression contains the quotient 8 ÷ 2, the possible results are 2 and 6. If the expression contains the quotient 6 ÷ 2, the possible results are −1, 7 and 9. If the expression contains the quotient 4 ÷ 2, the possible results are 0, 4 and 12. The list of all possible integer results, then, is −2, −1, 0, 2, 4, 6, 7, 9 and 12. The difference between the greatest and least of these results is 12 − (−2) = ***14****.*

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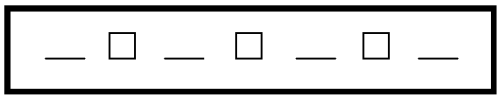

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