MATHCOUNTS® Problem of the Week Archive

Preparing for State – February 25, 2019

Problems & Solutions

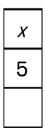
In just a few days, the 2019 MATHCOUNTS State Competitions will begin, so let's look back at some of the 2018 State Sprint round problems and solve them in preparation.

	x	
	5	
		$x+1$

An ordinary 3-by-3 magic square contains every positive integer from 1 through 9, with one integer per cell, such that the sums of the numbers in each row, each column and each diagonal are the same. When the ordinary magic square shown is completed, what is the sum of all the possible values of x?

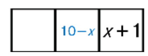
[Sprint #10]

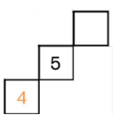
The sum of the elements in each row, column and diagonal of a 3-by-3 magic square must equal (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)/3 = (9 × 10)/(2 × 3) = 15.



For the values in the center column to add to 15, the bottom cell must have value 15 – (x + 5) = 15 – x – 5 = 10 – x.

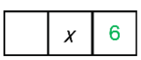
Then, for the values in the bottom row to add to 15, the left cell must have value 15 – (10 – x + x + 1) = 15 – 11 = 4.





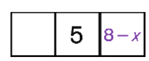
Next, for the values along the diagonal from the bottom-left to the top-right to add to 15, the top-right cell must have value 15 – (4 + 5) = 15 – 9 = 6.

Then for the values in the right column to add to 15, the middle right cell must have value 15 – (6 + x + 1) = 15 – 7 – x = 8 – x.



Next, for the values in the top row to add to 15, the left cell must have value 15 – (x + 6) = 15 – x – 6 = 9 – x.

Lastly, for the cells of the middle row to add to 15, the left cell must have value 15 – (5 + 8 – x) = 15 – 13 + x = x + 2.



Now we see three sets of 3 consecutive values: x, x + 1, x + 2; and 4, 5, 6; and 8 – x, 9 – x, 10 – x. Either one of the sets containing the unknown must be 1, 2, 3 and the other must be 7, 8, 9. Since x is the least value in its set, it follows that x can be either 1 or 7, the sum of which is 1 + 7 = ***8****.*

The team had an easy trip to the MATHCOUNTS contest, but a detour on the way home made the return trip take twice as long. If the new route home (including the detour) was 50% longer than the original trip, and the average speed returning was 10 mi/h slower, what was the average speed of the team going to the contest?

[Sprint #15]

Let d be the distance to the contest and t be the time to travel to the contest. Then the average speed going to the contest is d/t, which I will call v (for the magnitude of the velocity). Returning home involves a distance 50 % greater than going, so (1 + 0.5)d = 1.5d and a time twice as long as going, so 2t, making the average return speed 1.5d/2t = (1.5/2)(d/t) = (3/4)v, which is given to be 10 mi/h less than v. Therefore, (3/4)v = v –10 and 10 = v – (3/4)v = v/4, so v = 4 ×10 = ***40*** *mi/h.*

Bryan visits a carnival booth where Carl shows him 10 boxes. Exactly one of the boxes contains a gold coin; the other boxes are empty. Bryan randomly takes one of the boxes, but he doesn't open it. Carl then opens five other boxes that he knows are empty and shows Bryan that they are empty. Carl then tells Bryan he can either keep his initially chosen box or return it and choose one of the remaining closed boxes instead. If Bryan chooses to return his box and choose another one instead, what is the probability Bryan will choose the box with the gold coin? Express your answer as a common fraction.

[Sprint #23]

This is a variant of the Monty Hall problem. When Bryan firsts picks a box, the probability of him picking the one with the coin is 1/10 and of not picking the one with the coin is 9/10. After 5 boxes are removed, there are 5 left. In the 1/10 case, Bryan had the good box and a change would have 0 probability of yielding the good box. In the 9/10 case, Bryan does not have the good box, and the coin is equally likely to be in any one of the other 4 boxes, thus probability 1/4 to change to the good box. Thus, the probability of changing to the good box is (1/10) × 0 + (9/10) × 1/4 = ***9/40****.*

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