

MATHCOUNTS® Problem of the Week Archive

Space Junk – February 18, 2019

Problems & Solutions

Space junk is debris orbiting Earth that, for the most part, has been abandoned or has broken off spacecrafts and rockets. Some of these items are far enough from Earth that they will remain in orbit forever; however, much of this “junk” will be pulled into Earth’s atmosphere by gravity. On average, one item per day falls victim to gravity. At this rate, in what year will 2500 pieces of space junk enter the atmosphere if we begin counting on February 18th?

Since, on average, one piece of space junk enters Earth’s atmosphere per day, including what falls on February 18th, a total of $365 - (31 + 17) = 365 - 48 = 317$ more pieces will fall to Earth in 2019. So, we need to figure out by what year another $2500 - 317 = 2183$ pieces will have fallen. At a rate of 365 pieces per year (366 pieces per leap year), in the five years from 2020 to 2024, another $366 + 365 + 365 + 365 + 366 = 366 \times 2 + 365 \times 3 = 732 + 1095 = 1827$ pieces of space junk will fall to Earth. The remaining $2183 - 1827 = 356$ pieces (to reach a total of 2500 fallen pieces) will fall to Earth the following year, in **2025**.

Two pieces of space junk are following the same orbit path, one is 1000 miles behind the other. If the front piece of space junk is traveling at a speed of 17,000 miles per hour and the other piece of space junk is traveling 17,500 miles per hour, how many miles will the front piece of junk travel before the second piece catches up to it?

To solve this problem, we will use the formula $\text{time} = \text{distance}/\text{rate}$. At the point that the two pieces of space junk meet, both will have traveled for the same length of time, but the piece of junk that started behind the first will have traveled 1000 miles farther. Based on this, we have these two equations for time t and distance d : $t = d/17,000$ and $t = (d + 1000)/17,500$. Setting the expressions for t equal to each other gives us $d/17,000 = (d + 1000)/17,500$. Cross-multiplying and solving for d , we see that $17,500d = 17,000(d + 1000) \rightarrow 175d = 170d + 170,000 \rightarrow 5d = 170,000 \rightarrow d = \mathbf{34,000}$ miles is the distance the front piece of space junk travels before the second piece catches up to it.

A piece of space junk is orbiting Earth, traveling 16,500 miles per hour. If the piece of space junk is 8550 miles above Earth’s center, how many hours does it take this piece of space junk to make one complete orbit around Earth? Express your answer as a decimal to the nearest tenth.

First, we need to find out the length of the space junk’s orbit, which is a circle of radius 8550 miles. We have $C = 2\pi r = 2\pi(8550) = 17,100\pi$ miles. Since $\text{time} = \text{distance}/\text{rate}$, the time it takes this piece of space junk to make one complete orbit around Earth is $17,100\pi/16,500 \approx \mathbf{3.3}$ hours.

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