

# MATHCOUNTS® Problem of the Week Archive

## Sequences and Patterns – February 5, 2018

### Problems & Solutions

This Saturday's date, 02-10-18, forms an arithmetic sequence since  $2 + 8 = 10$  and  $10 + 8 = 18$ . In honor of this occurrence, try your hand at solving these problems involving sequences and patterns.

What is the tenth term of the sequence  $-50, -49, -47, -44, -40, \dots$ ?

*The difference between consecutive terms in this sequence is not constant since  $-50 - (-49) = -1$ ,  $-49 - (-47) = -2$ ,  $-47 - (-44) = -3$  and  $-44 - (-40) = -4$ . Notice, though, that there is a pattern with the differences between consecutive terms, which are  $-1, -2, -3, -4$ . So, if the pattern continues, the next five terms in this sequence will be  $-35, -29, -22, -14$  and  $-5$ . Thus, the tenth term in the sequence is **-5**.*

What is the sixth term in the sequence  $0, 3, 8, 15, 24, \dots$ ?

*Each term equals a perfect square minus one. The first term is  $1^2 - 1 = 0$ . The second term is  $2^2 - 1 = 3$ . The third term is  $3^2 - 1 = 8$ . The fourth term is  $4^2 - 1 = 15$ . The fifth term is  $5^2 - 1 = 24$ . Therefore, the sixth term is  $6^2 - 1 = \mathbf{35}$ .*

Given the list of numbers  $3, 2, 6, 3, 9, 5, 12, 7, \dots$ , if the pattern continues what is the sum of the ninth and tenth numbers?

*There are two patterns occurring within the list of numbers in this question, one for the odd-numbered terms and another for the even-numbered terms. First, the first, third, fifth and seventh numbers are  $3, 6, 9$  and  $12$ . Notice that these numbers are all multiples of three ( $3 \times 1, 3 \times 2, 3 \times 3$  and  $3 \times 4$ ). So, the ninth number will be  $3 \times 5 = 15$ . Next, the second, fourth, sixth and eighth numbers are  $2, 3, 5$  and  $7$ . Notice that these are positive prime numbers listed in order. So, the tenth number will be  $11$ . The sum of the ninth and tenth numbers is  $15 + 11 = \mathbf{26}$ .*

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### *Problems*

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