

# MATHCOUNTS® Problem of the Week Archive

***Time to Compete! – January 28, 2019***

## ***Problems & Solutions***

Chapter competitions officially start this week! Are you ready to compete? Perhaps solving these Sprint Round problems from the 2018 Chapter Competition will help you warm up.

### Sprint #15

By switching two of the digits of the number 123,456, Rodrigo obtains a new number that is 1980 more than the original. What is the product of the two digits Rodrigo switched?

*Adding 1980 to 123,456 gives us  $123,456 + 1980 = 125,436$ . We see that the digits that have been switched are 3 and 5. The product is  $3 \times 5 = 15$ .*

### Sprint #17

Dak has a quarter, a dime, a nickel and a penny. How many different amounts can be obtained by using one or more of the coins in Dak's collection?

*The values of 25¢, 10¢, 5¢ and 1¢ are separated enough that duplicated sums cannot occur. In any possible combination of one or more of the coins, the quarter can either be included or excluded (2 options), likewise for the dime (2 options), the nickel (2 options), and the penny (2 options). Each of these four inclusion-exclusion cases is independent of the others, so there are  $2 \times 2 \times 2 \times 2 = 16$  possible combinations of coins. However, this includes the outcome where all four coins are excluded, which does not use one or more of the coins. Therefore, there are  $16 - 1 = 15$  possible amounts.*

### Sprint #27

For a particular list of four distinct integers the mean, median and range have the same value. If the least integer in the list is 10, what is the greatest value for an integer in the list?

*Let the four distinct values be  $10 \leq a \leq b \leq c$ . The mean is  $(10 + a + b + c)/4$ . The median is  $(a + b)/2$ . The range is  $c - 10$ . The mean being equal to the median implies  $(10 + a + b + c)/4 = (a + b)/2 \rightarrow 2(10 + a + b + c) = 4(a + b) \rightarrow 10 + a + b + c = 2a + 2b \rightarrow 10 + c = a + b$ . The median being equal to the range implies  $(a + b)/2 = c - 10 \rightarrow a + b = 2(c - 10) \rightarrow a + b = 2c - 20$ . Substituting  $2c - 20$  for  $a + b$  in the equation  $10 + c = a + b$  yields  $10 + c = 2c - 20$ . Solving, we get  $c = 30$ .*

# **MATHCOUNTS<sup>®</sup> Problem of the Week Archive**

***Time to Compete! – January 28, 2019***

## ***Problems***

Chapter competitions officially start this week! Are you ready to compete? Perhaps solving these Sprint Round problems from the 2018 Chapter Competition will help you warm up.

### Sprint #15

By switching two of the digits of the number 123,456, Rodrigo obtains a new number that is 1980 more than the original. What is the product of the two digits Rodrigo switched?

### Sprint #17

Dak has a quarter, a dime, a nickel and a penny. How many different amounts can be obtained by using one or more of the coins in Dak's collection?

### Sprint #27

For a particular list of four distinct integers the mean, median and range have the same value. If the least integer in the list is 10, what is the greatest value for an integer in the list?