MATHCOUNTS® Problem of the Week Archive

Happy New Year! – January 7, 2019

Problems & Solutions

Happy New Year! Now that 2019 is here, let's have some fun solving this collection of problems that all involve the number 2019.

What is the greatest prime factor of 2019?

*The factors of 2019 are 1, 3, 673, 2019. The prime factors of 2019 are 3 and 673, the greatest being **673**.*

Consider all 365 dates of 2019 expressed in the form MM-DD-YY, where MM, DD and YY represent the two-digit month, two-digit day and two-digit year, respectively. For how many of these dates is the sequence MM, DD, YY an arithmetic sequence?

*Since all the dates are in the year 2019, we are looking for sequences of the form MM, DD, 19. In order for the numbers to form an arithmetic sequence, the absolute difference between consecutive terms must be constant. In other words, DD − MM = 19 − MM, so the value of DD must be half-way between the value of MM and 19. That requires the difference between 19 and MM to be even (we must be able to divide it in half). The difference 19 − MM is even only when the value of MM is odd. This is the case for half of the 12 months of 2019, so we conclude that 01, 10, 19; 03, 11, 19; 05, 12, 19; 07, 13, 19; 09, 14, 19; and 11, 15, 19 are the **6** dates for which MM, DD, YY is an arithmetic sequence.*

What is the 2019th term of an arithmetic sequence in which the 3rd term is 1989 and the 8th term is 2019?

*For this particular arithmetic sequence, to get from the 3rd term to the 8th term, the common difference d must be added 8 − 3 = 5 times. So, $1989 + 5d = 2019 \rightarrow 5d = 30 \rightarrow d = 6$. Now, to get from the 8th term to the 2019th term, the common difference d = 6 must be added 2019 − 8 = 2011 times. The 2019th term of this sequence, then, is 2019 + 2011(6) = **14,085**.*

The number 2019 can be expressed as the difference of the squares of positive integers *a* and *b*. What is the sum of all the possible values of *a* and *b*?

*From the problem statement we know that $a^2 - b^2 = 2019$. Since $a^2 - b^2 = (a - b)(a + b)$, it follows that $(a - b)(a + b) = 2019$. If we factor 2019, we find that there are only two positive integer factor pairs that have a product of 2019: 1 x 2019 and 3 x 673. Let's first look at the case when $(a - b)(a + b) = 1 \times 2019 = 2019$. If a − b = 1 and a + b = 2019, then a and b must be consecutive. Since 2019/2 = 1009.5, let's try a = 1010 and b = 1009. We get $(a - b)(a + b) = (1010 - 1009)(1010 + 1009) = (1)(2019) = 2019$. Using a calculator, we can quickly verify that $1010^2 - 1009^2$ does equal 2019. Now let's examine the case when $(a - b)(a + b) = 3 \times 673 = 2019$. If a − b = 3 and a + b = 673, then a and b are close (difference of 3) but not consecutive. Since 673/2 = 336.5, let's start by making a = 337 and b = 336. But since the difference between them is 3, a needs to be more than 337 and b needs to be less than 336. But remember they still need to add to 673. If a = 338 and b = 335, then a − b = 338 − 335 = 3 and a + b = 338 + 335 = 673. So we have $(a - b)(a + b) = (338 - 335)(338 + 335) = (3)(673) = 2019$. Again, a calculator helps to quickly verify that $338^2 - 335^2$ does equal 2019. Since we are asked to find the sum of all possible values of a and b, the answer is 1010 + 1009 + 338 + 335 = 2019 + 673 = **2692**.*

Happy New Year! – January 7, 2019

Problems

Happy New Year! Now that 2019 is here, let's have some fun solving this collection of problems that all involve the number 2019.

What is the greatest prime factor of 2019?

Consider all 365 dates of 2019 expressed in the form MM-DD-YY, where MM, DD and YY represent the two-digit month, two-digit day and two-digit year, respectively. For how many of these dates is the sequence MM, DD, YY an arithmetic sequence?

What is the 2019th term of an arithmetic sequence in which the 3rd term is 1989 and the 8th term is 2019?

The number 2019 can be expressed as the difference of the squares of positive integers a and b. What is the sum of all the possible values of a and b?