THE TRANSFORMATION GAME

Everything You Need to Play

Materials

Computer with Internet access

- 1 computer per pair of students.
- If you do not have computers with Internet access, students can play using a printed game board and pencil.

Score Sheets & Writing Utensils

- 1 per student—to track each player's moves and points earned.

Desmos Geometry Instructional Video & Game Boards

- Found at https://www.mathcounts.org/resources/transformation-game. Students should watch the instructional video before playing.
- Links to ten different game boards are available for students to use.

Rules

The Transformation Game allows students to practice with four different geometric transformations in the coordinate plane: translations, reflections, rotations and dilations. The Transformation Game boards are *xy*-coordinate grids including two or more shapes. Students will play one-on-one (or collaboratively) to try to map the pre-image onto the image using the fewest number of moves.

- Decide who will be Player A and who will be Player B—rock-paper-scissors, flip a coin, etc.
- To start, Player A transforms the pre-image onto the image in as few moves as possible (the pre-image is always blue and the image is always green).
- Player B should not observe Player A's moves so that he/she approaches the same game board with no advantage.
- Moves are defined as follows:
  - – Each ***reflection*** over a line counts as 1 move.
  - – Each ***rotation*** counts as 1 move (e.g. a 90-degree rotation counts as 1 move, but if a player finds he/she needs to rotate the shape again by 45 degrees, this counts as a second move).
  - – Each ***dilation*** counts as 1 move (e.g. a dilation by a factor of 5 counts as 1 move, but if a player finds he/she overestimated and needs to dilate again by a factor of 4/5, this counts as a second move).
  - – Each unit of ***translation*** counts as that many moves (e.g. if a player translates a shape 10 units up, this counts as 10 moves).
- The number of moves dictates the number of points a player has. Player A should record what each move was on his/her score sheet in the "Moves" column (e.g. Rotation of 90 degrees counterclockwise) as well as the number of points received as a result of that move

(e.g. 1). The goal is to end with the fewest number of points.

- Player A finishes his/her turn by writing down the total number of moves he/she used to map the pre-image onto the image. This is his/her final score.
- Player B resets the same grid and shapes by reopening the same Desmos link or by undoing all of the moves made by Player A. Player B attempts to map the pre-image onto the image using fewer moves than Player A. Player B records his/her moves and points on his/her score sheet.
- The player with the fewest number of points wins the round. Players can play for just one round or decide that multiple rounds make up a full game, with the winning player having accumulated the fewest number of points in total. MATHCOUNTS has provided ten pre-made game boards, but feel free to make your own as well at https://www.desmos.com/geometry.

Additional Rules (Optional)

- Players may not undo moves. Once a move has been made, that is the starting point for the next move. If they choose to reverse a previous move, it must count toward the final score.
- Without significant additional constructions, some specific moves might be difficult to accomplish. If a player attempts a transformation that results in the shape being slightly off of the intended placement due to technical error, not mathematical error, you may choose to have this count as accurate or to allow players to re-try the move to make it perfect. For example, attempting to eyeball the placement of a line of reflection exactly halfway between two grid lines, may not be exact.

Differentiation, Scaling & Extensions

Change the Rules

Limit the shapes and transformations allowed.

- Only allow students to use game boards with simple polygons to keep the game at an easier level. Similarly, only allow students to use game boards with a single shape or that only require certain transformations.
- Require a different transformation in each round. For example, you might say that no matter which game board students have, they must apply a rotation for at least one of their moves.
- Change the point system. Make certain transformations worth more points, for example, or make a single translation of however many units be worth only one point.

Make a New Board

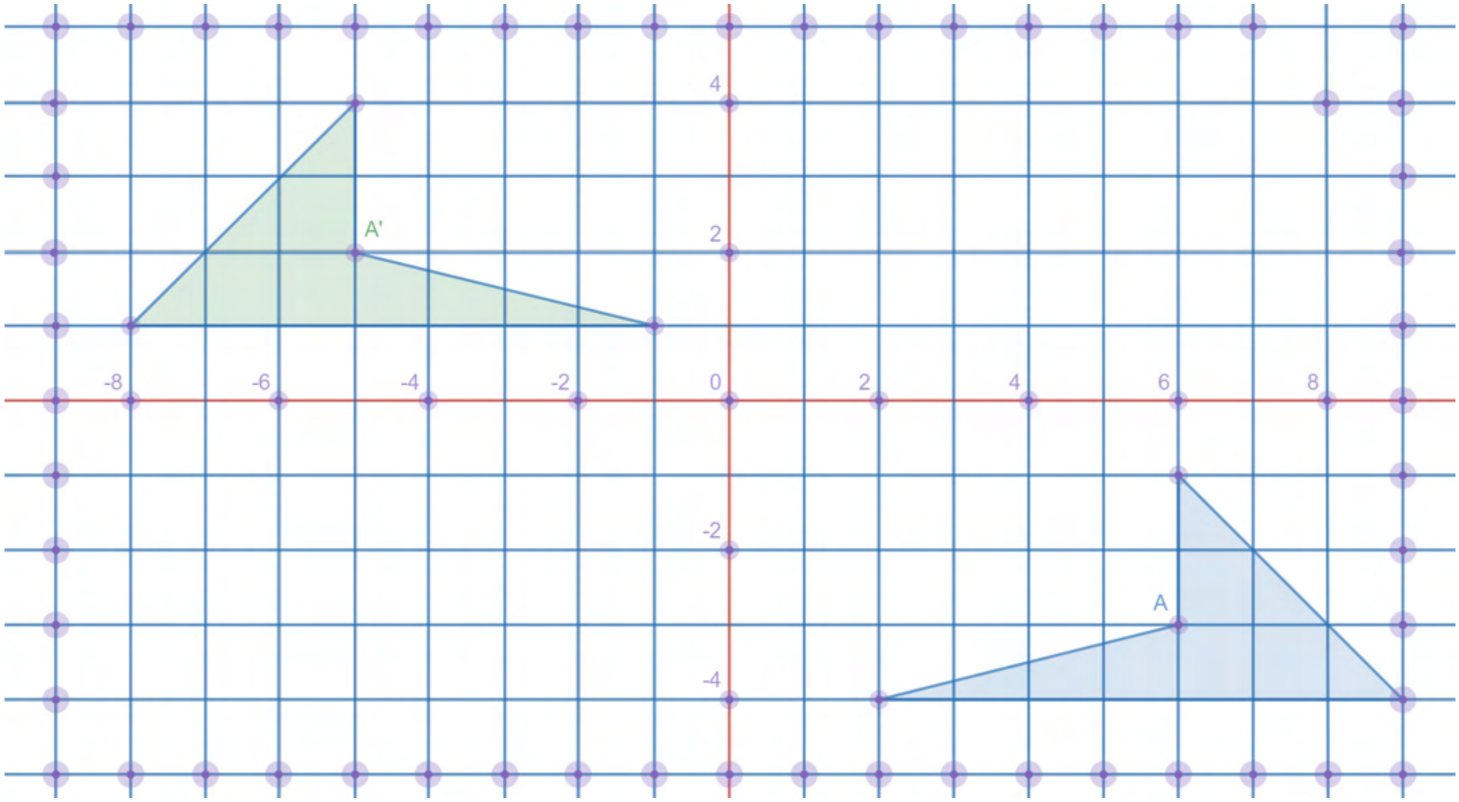
Allow students to create their own boards at https://www.desmos.com/geometry. They'll need to create a pre-image, an image, axes and grid lines. Alternatively, students can leave out the axes and grid lines and simply play on a blank grid. Note, however, that this may change the steps needed to perform transformations in Desmos throughout the game. For example, if a player wants to perform a reflection without axes and grid lines pre-created, the player will first need to create the desired line of reflection on the board before being able to apply a reflection over this line.

THE TRANSFORMATION GAME

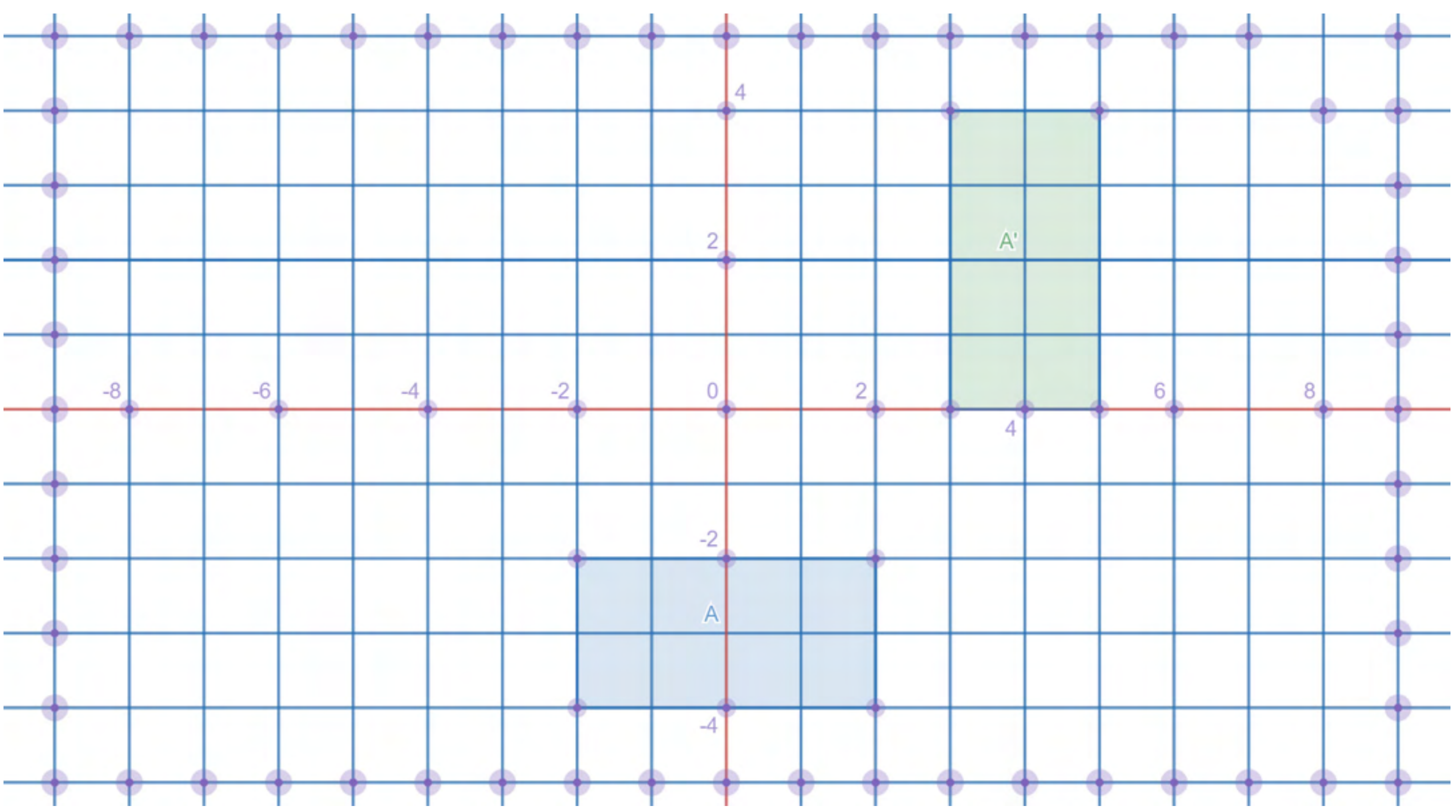
Score Sheet

Moves	Points
1. ______________	1. ______________
2. ______________	2. ______________
3. ______________	3. ______________
4. ______________	4. ______________
5. ______________	5. ______________
6. ______________	6. ______________
	Round Total:

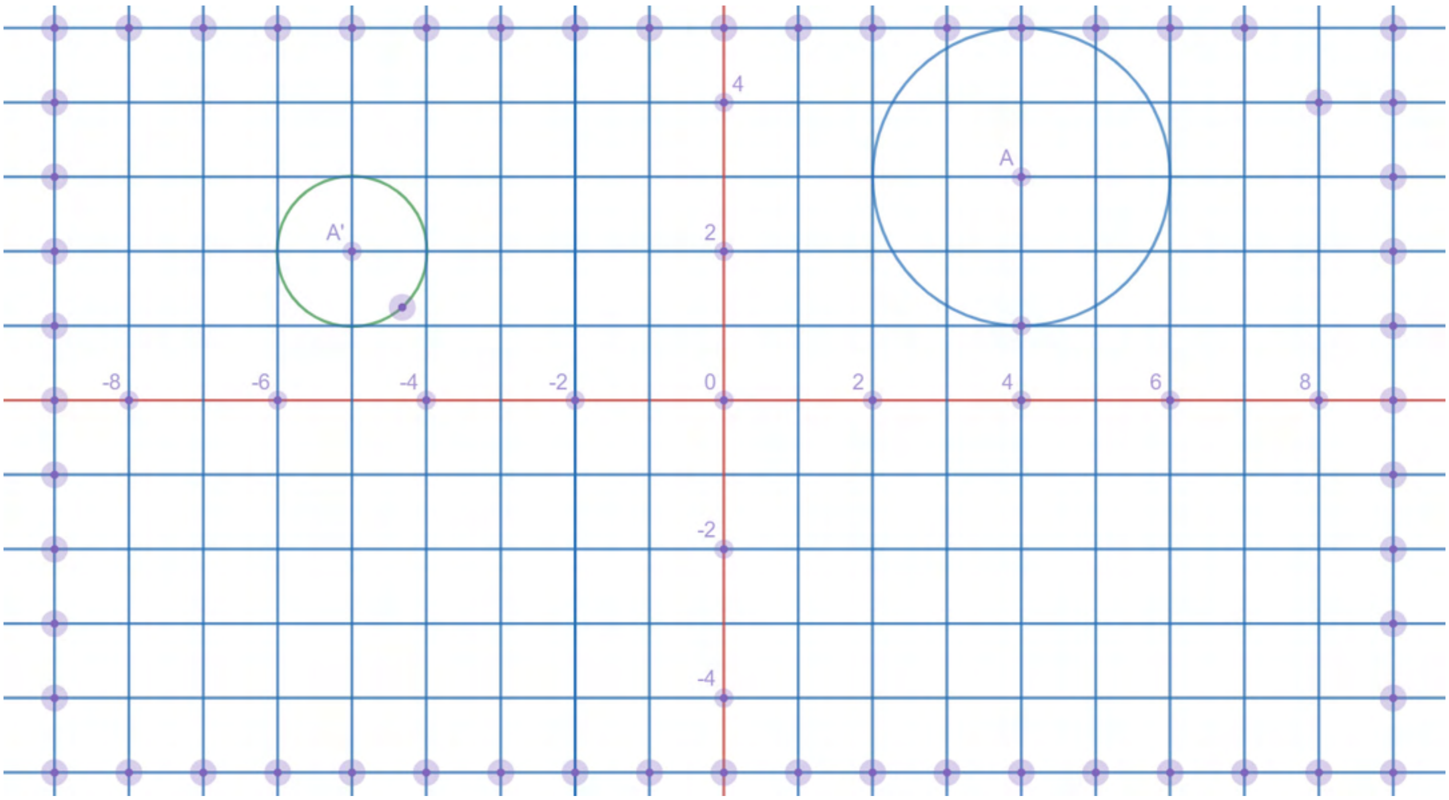
Board 1



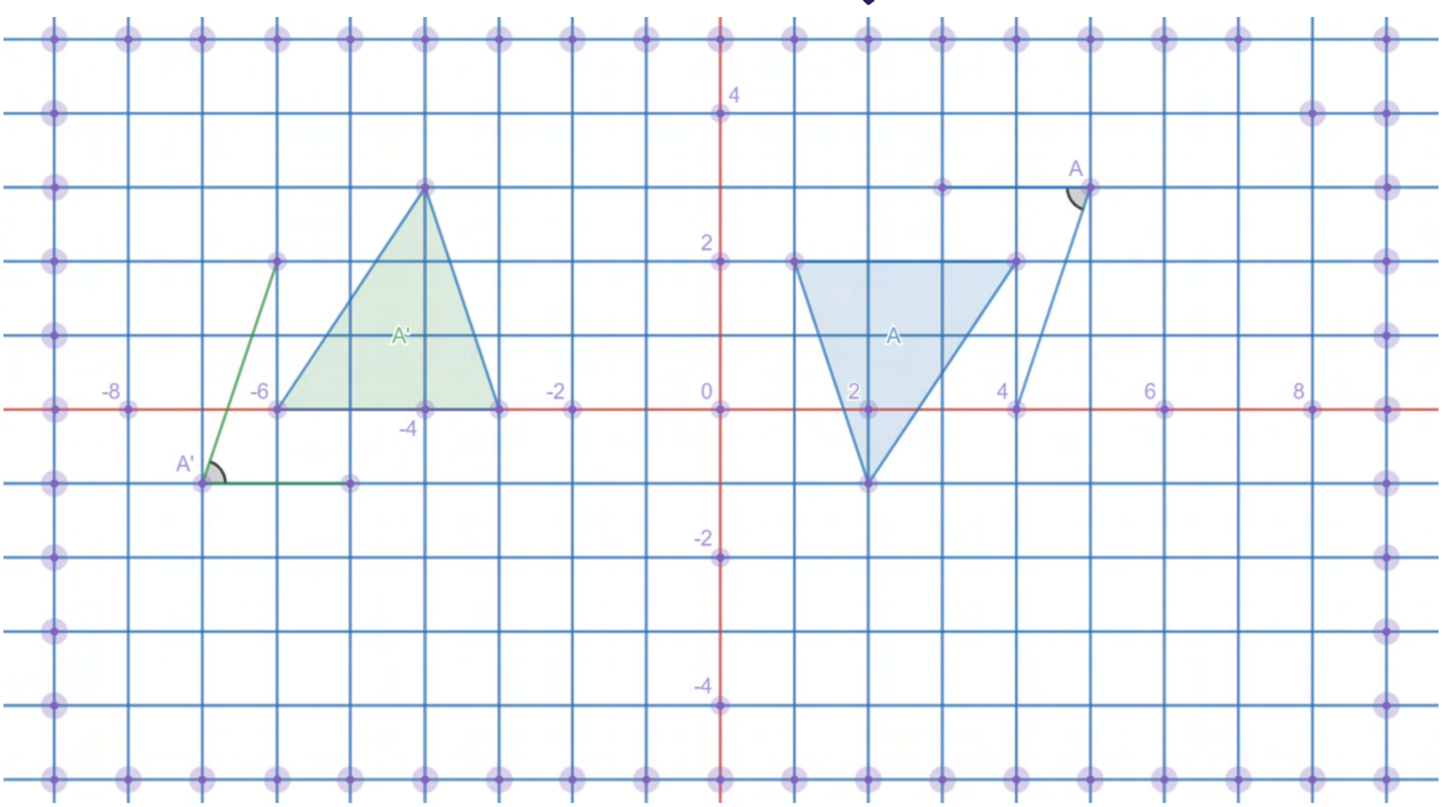
Board 2



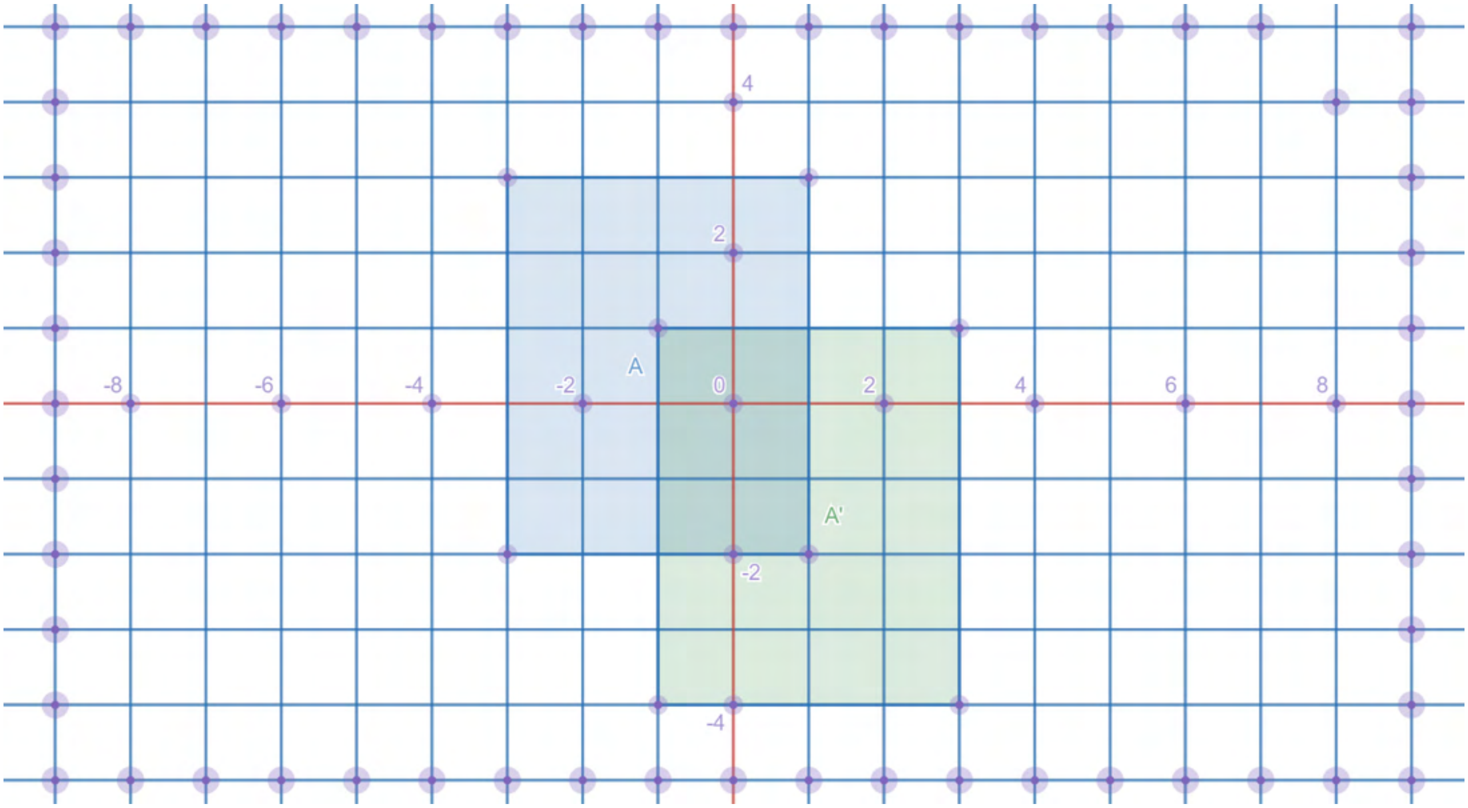
Board 3



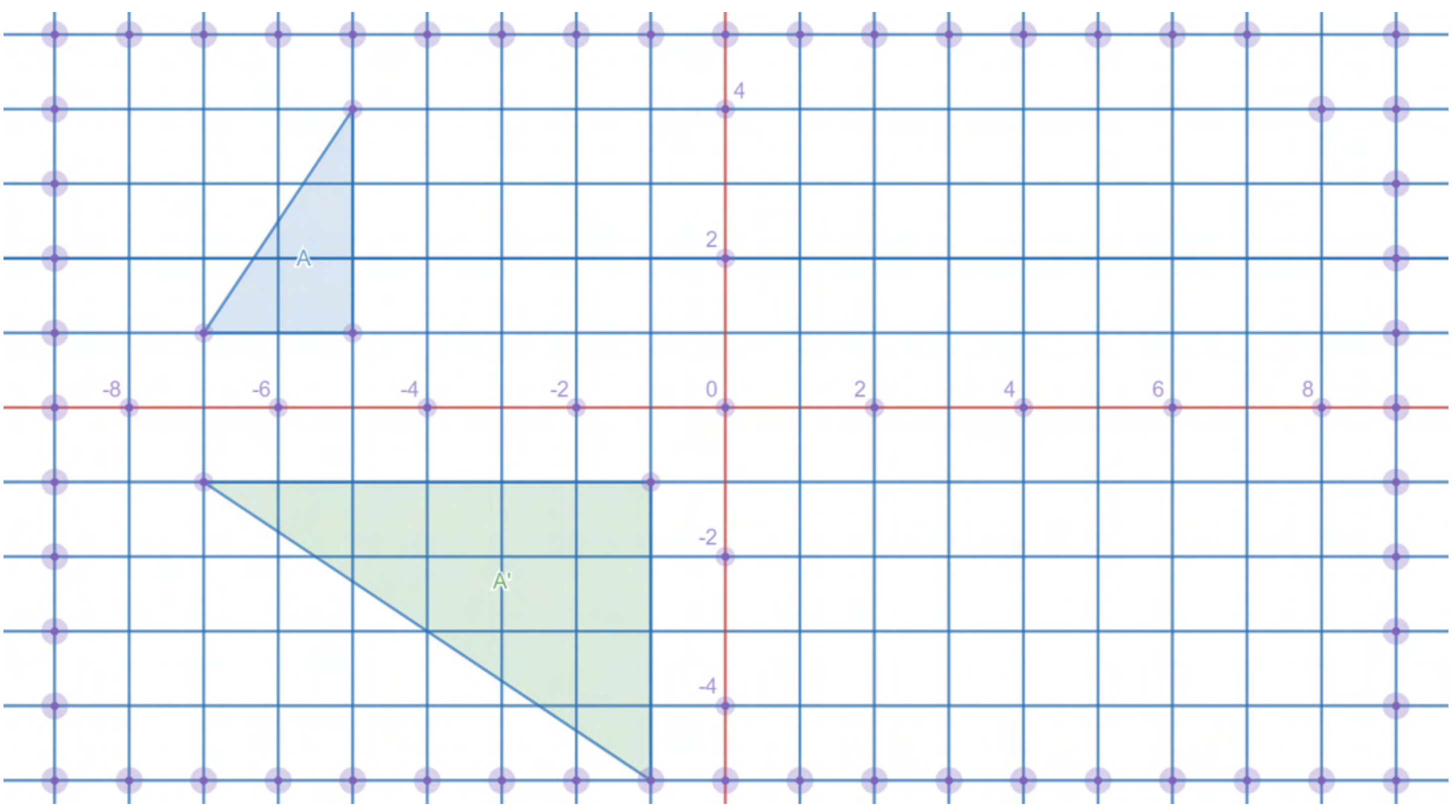
Board 4



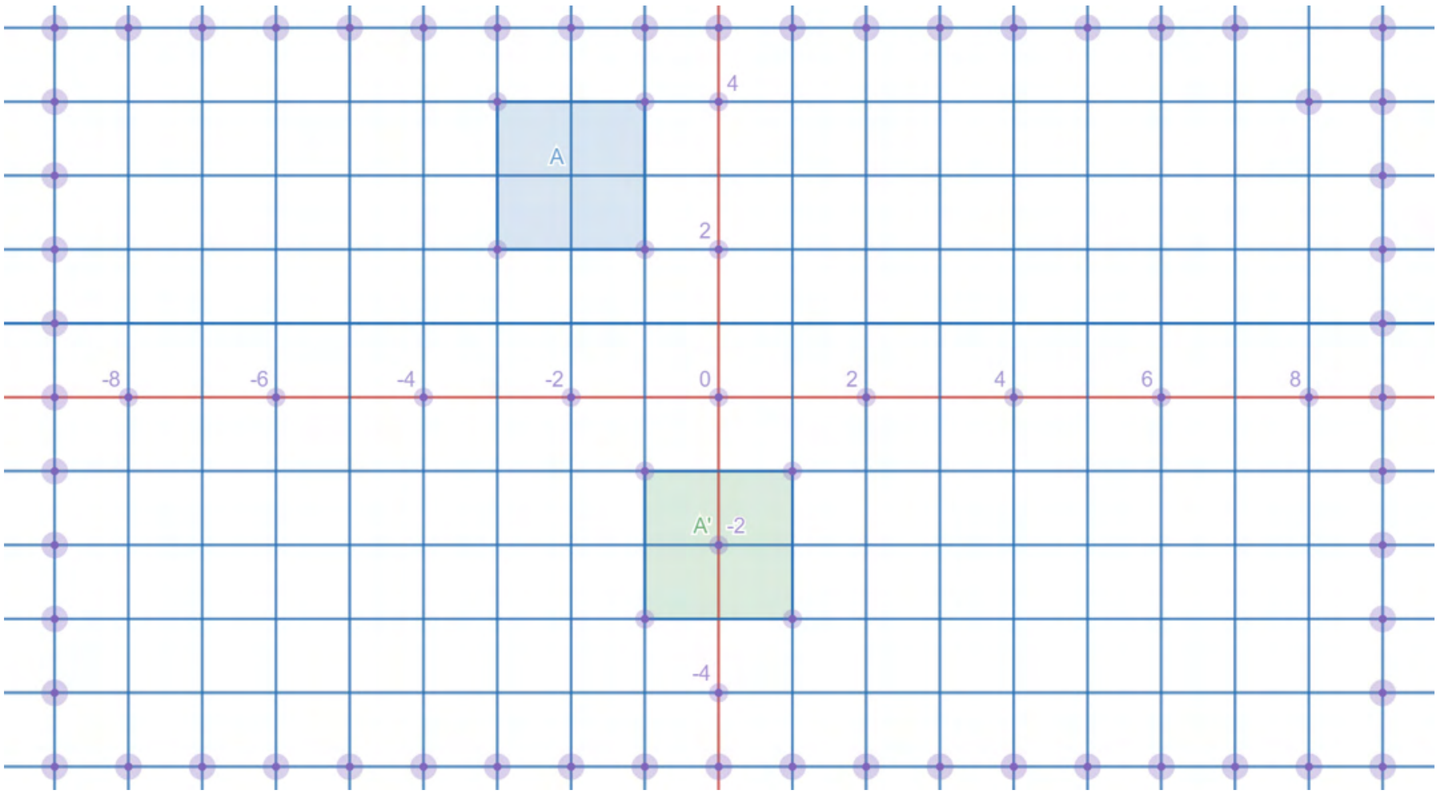
Board 5



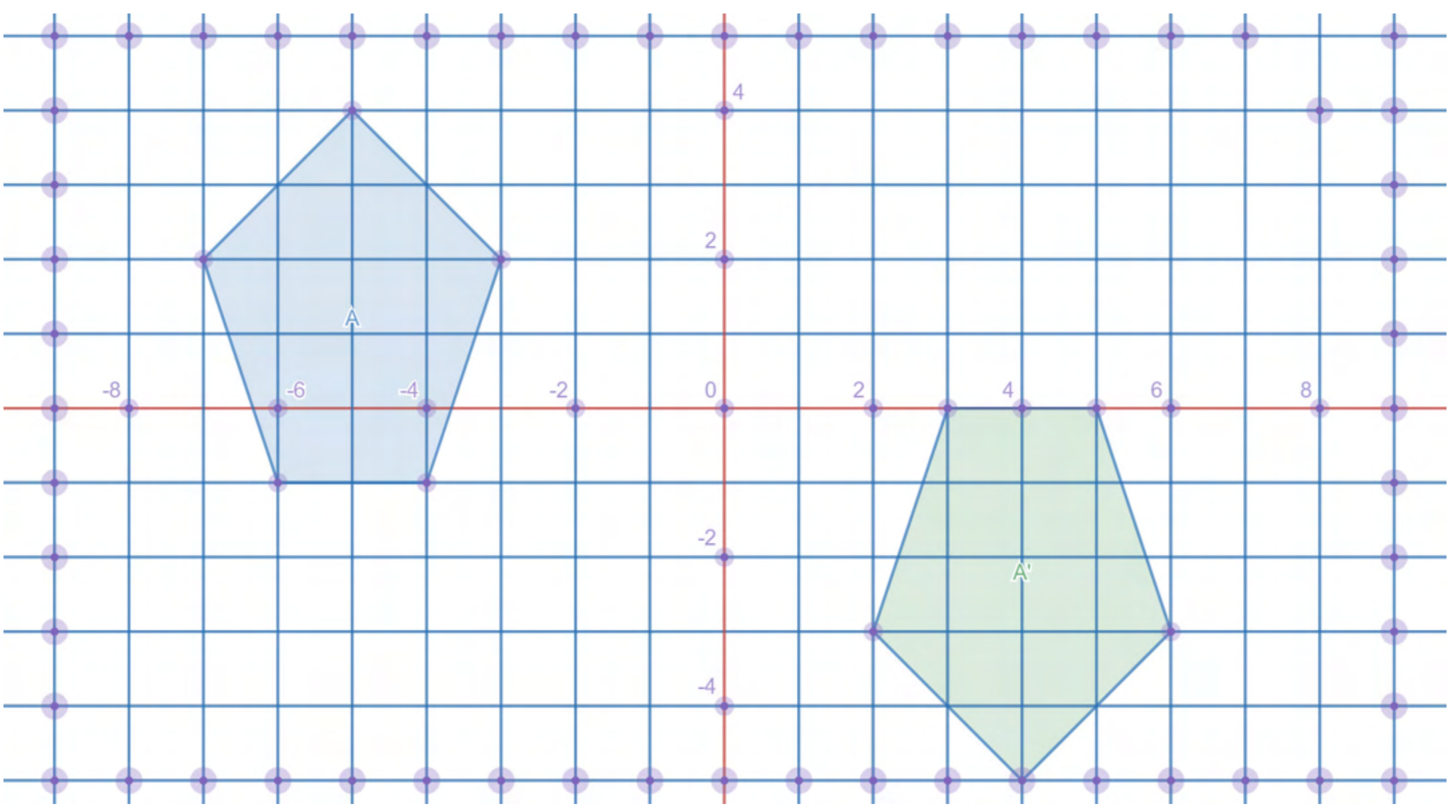
Board 6



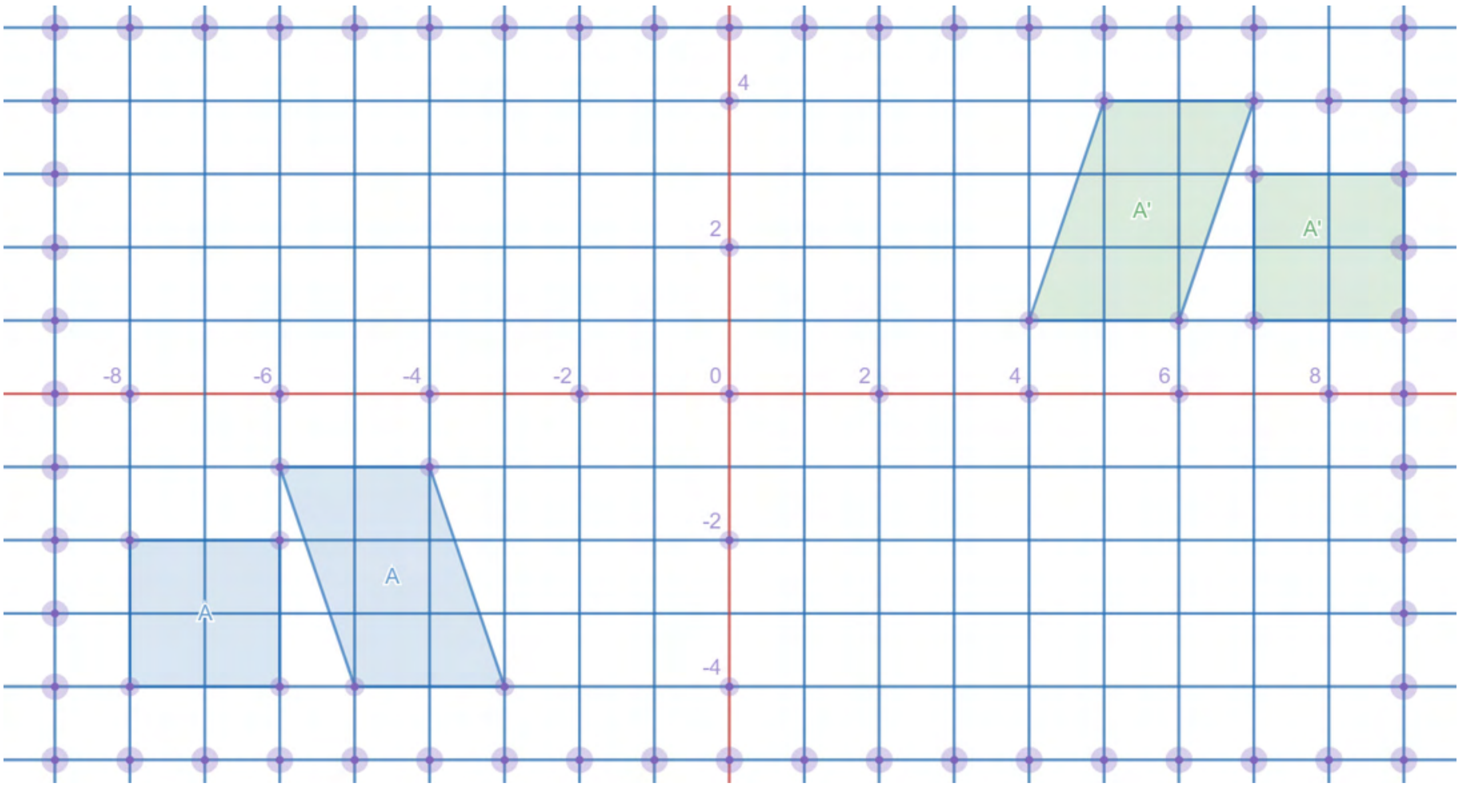
Board 7



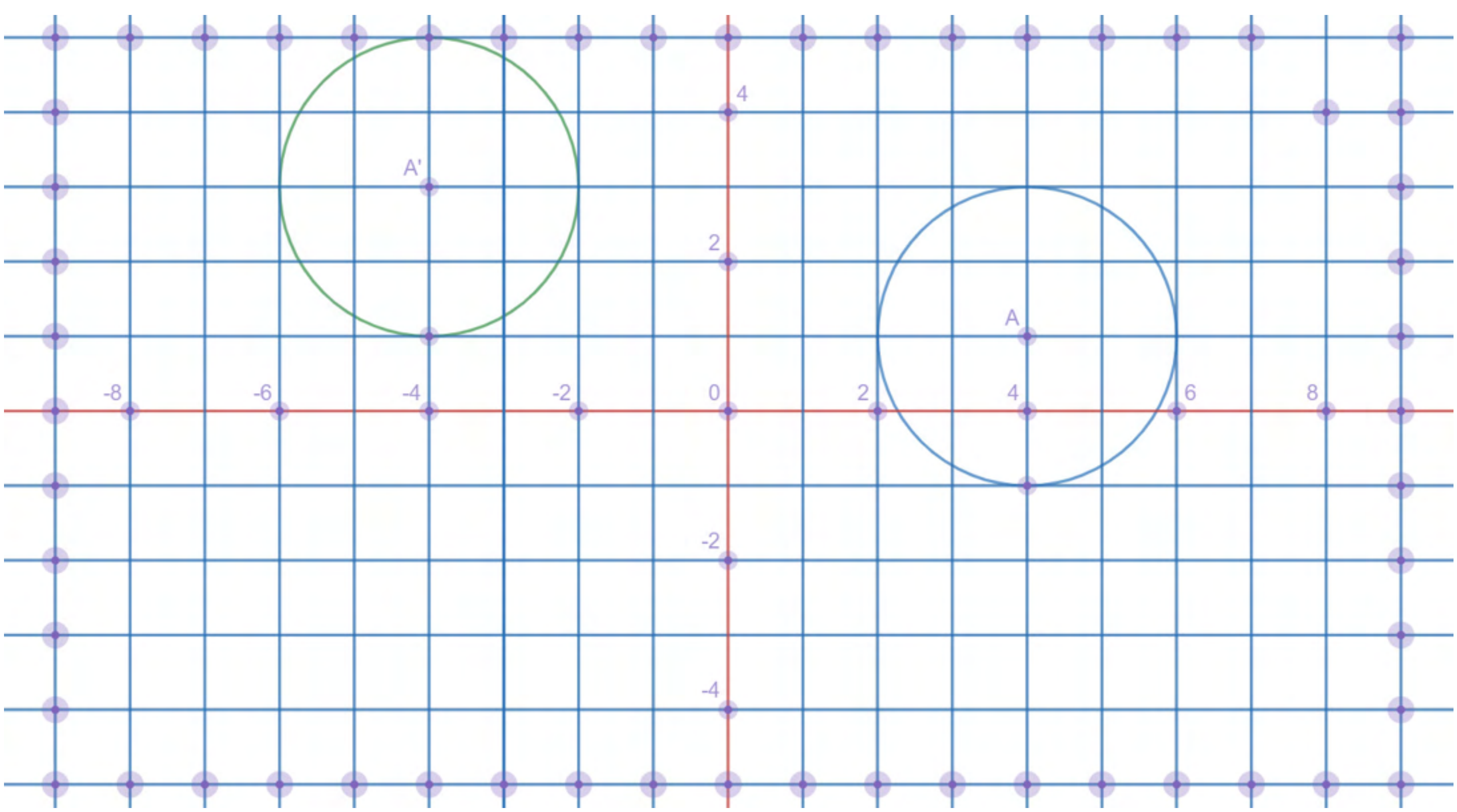
Board 8



Board 9



Board 10



THE TRANSFORMATION GAME

Mathematical Exploration

Choosing Transformations

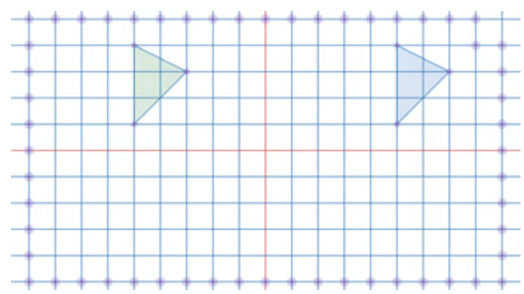
Often students are asked to perform a transformation on a shape. This game forces them to narrow their scope to determine which transformation(s) would map one shape onto another, existing shape. This section explores how to use properties of a pre-image and an image to determine which transformations are necessary and/or possible and which will result in the fewest number of moves from one shape to the other.

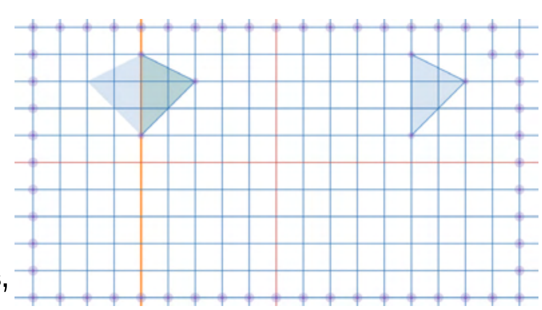
- ***When do you need to perform a dilation?***

  A dilation keeps the proportions of a shape the same but changes its size (with the exception of a dilation by a scale factor of 1). So, when playing The Transformation Game, it's important to look at the provided pre-image and image together. If they are the same size, no dilation is necessary! If they are not the same size, however, you will need to determine by what factor the shape has grown or shrunk. For polygons, it is easiest to find the length of a side of the shape that is perfectly vertical or horizontal along a grid line. Count the units along the length of this side in the pre-image. Then, locate the corresponding side in the image, and count the units along that side. How do the lengths compare? If, for example, the length of a side in the pre-image is 2 units in length, and the length of the corresponding side in the image is 6 units in length, you will need to perform a dilation by a scale factor of 3 in order to make the shapes the same size. Similarly, with circles, you can find the length of the diameter in both the pre-image and the image to determine the needed scale factor. Another note on dilations: there are many possible centers of dilations, so pay attention to which one you're choosing. The center of dilation determines where the shape ends up after you perform the dilation!

- ***When do you need to perform a translation?***

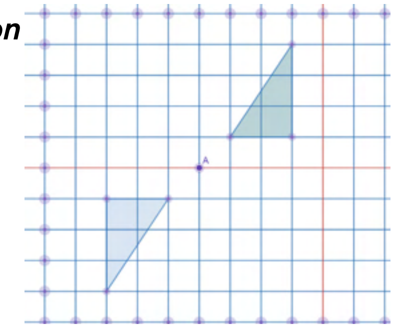
  A translation simply moves a shape. It does not change the orientation or size of a shape in any way. As you're playing The Transformation Game, if an image is oriented in exactly the same way as the pre-image, a translation could take you there. Be careful, though! Translations give you more points than you may need. Sometimes, a combination of rotations and/or reflections can result in an image that is oriented in the same way as the pre-image. Depending on the exact locations of the shapes, rotations and/or reflections might be more efficient than translations. For example, in the figure at the top right, a translation of 10 units to the left would accurately map the pre-image onto the image. However, a reflection of the pre-image over the *y*-axis, followed by a reflection over the orange line results in the image in fewer moves (as seen at the bottom right).

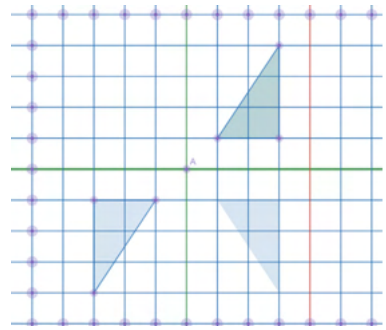


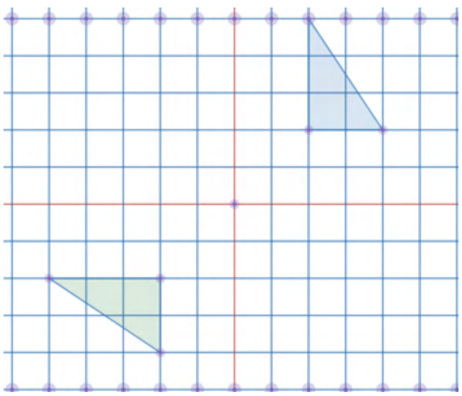


How can you tell if you need to perform a reflection, a rotation or a combination?

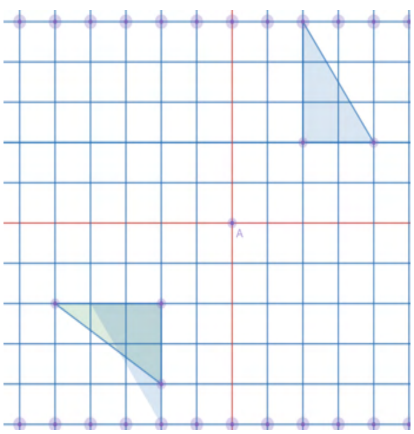
This is where things get a bit tricky. A reflection is a mirror image of a shape over a specified line of reflection, while a rotation moves a shape in a circular motion around a specified point for a specified number of degrees. Both of these transformations change the orientation of a shape, so a shape could end up "upside down," for example, from either a reflection or a rotation. So how do you know which one to choose? Let's look at a couple of examples, starting with the pre-image and image at right.







There are a couple of different ways to use reflections or rotations to map the pre-image (blue) onto the image (green). You can tell these transformations are both possibilities because of the orientation of the pre-image in relation to the orientation of the image. In this case, a 180-degree rotation around point A maps the pre-image onto the image, as seen in the figure at the top right. We can also see that a sequence of reflections could get us to the same place. Reflecting the pre-image over the vertical line through point A, then reflecting again over the horizontal line through point A maps the pre-image onto the image. In this case, performing a rotation would result in fewer moves.



Let's look at second example, using the pre-image and image at the top left. If we try to perform a 180-degree rotation around point A (the origin), the pre-image will end up in the correct quadrant, but does not map onto the image, as seen in the bottom left. Sometimes, performing rotations in 3-D, using paper cut-outs of the pre-image and image, can clarify whether or not shapes actually match up. Here, a single reflection over the diagonal line $y = -x$ will map the pre-image onto the image. You might also use a ***composite transformation*** (i.e. a combination or sequence of transformations). In this example, you can map the pre-image to the image by rotating the pre-image 90 degrees clockwise around point A, then reflecting over the y-axis. Reflections and rotations often work well together in composite transformations, so keep this in mind as you play!

With some shapes, like squares and circles, a reflection over a certain line and a rotation around a certain point could have an identical effect on the pre-image. However, more often than not, reflections and rotations will result in different images. Be sure you're getting a good look at the pre-image and the image before making any moves!