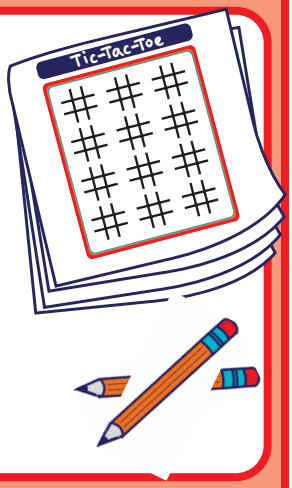
THREE TIC-TAC-TOES

EVERYTHING YOU NEED TO PLAY

MATERIALS



Tic-Tac-Toe Game Board

- 1 game board per pair of students. Find a copy of the game board below. This game board is easy to duplicate without the need to make additional copies—just use paper and pen or pencil.
- Using a paper copy of the board is fine, but you may also want to make a laminated copy for multiple uses. A sheet protector is an easy substitute for lamination!

Writing Utensils

- 1 per student—pencils, pens or dry erase markers (for laminated boards).

RULES

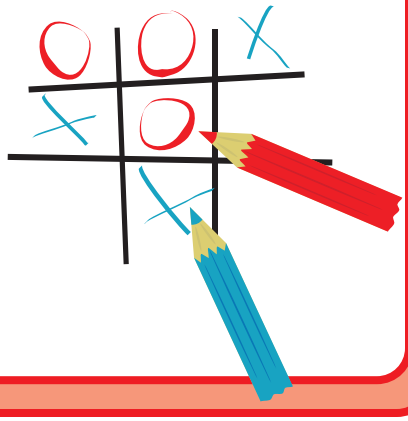
Chances are that most if not all of your students are familiar with tic-tac-toe. This game, although simple in its rules, has a lot of mathematical value. When students play the game, they are using logic and problem solving whether they know it or not. Play traditional tic-tac-toe with your students, and then challenge them with some rule variations that will keep them thinking!

Traditional Tic-Tac-Toe

- Decide who will be Player A and who will be Player B, as well as which player will be X's and which will be O's.
- To start, Player A places a mark in one of the 9 open spaces on the board.
- Next, Player B repeats the process but using the mark not used by Player A. Players alternate marking open spaces with X's or O's until one player has marked three in a row to win.
- A tie, or cat's game, is possible, in which case players can simply start a new game.

Misere Tic-Tac-Toe

- This game is played almost identically to traditional tic-tac-toe, but the goal is to NOT get three in a row.
- Each player should again be either X's or O's, and players alternate turns. This time, however, the goal is to force the OTHER player to mark three in a row. The player who marks three in a row first loses the game. There is also the option for a tie, or a cat's game.



Wild Tic-Tac-Toe

- In this game, players alternate turns with the same goal as the traditional game—to get three in a row.
- The difference for this version is that on each turn, a player can choose to place an X or an O. Players are not assigned one mark to use for the entire game.
- Players now must try to set themselves up for three in a row but

Optional: Make It Extra Mathy

Mathematical Exploration

Symmetry and Combinatorics

Although a seemingly simple game, tic-tac-toe can be explored with a lot of mathematics. For the purposes of brevity in this exploration section, we are going to explore concepts through the lens of the traditional game. Some of the answers to the questions posed will be true for all game variations, whereas others may require adjusting based on the modified rules, and you might like to go over them with your club.

- ***In tic-tac-toe, the board has 9 empty squares to begin the game. Does this mean there are 9 unique opening moves? If not, how many are there?***

  If students analyze the board, they might notice it is rotationally, vertically, horizontally and diagonally symmetric. Based on this symmetry, there are really only 3 opening moves. There are 4 corners, 4 sides and 1 center, and with only 1 move on the board, all corners are equivalent, as are all sides.

- ***Using the idea of board symmetry, how many unique second moves are there? Or in other words, how many ways are there for the first 2 moves of the game to play out?***

  Just as there appear to be 9 options for the first move, there then appear to be 8 for the second. In a standard combinatorics calculation, one might assume tic-tac-toe has $9 \times 8 = 72$ possible two-move starts. However, due to symmetry, this is not the case. Let's take our 3 unique opening moves and use our symmetry knowledge to determine the possible second moves.

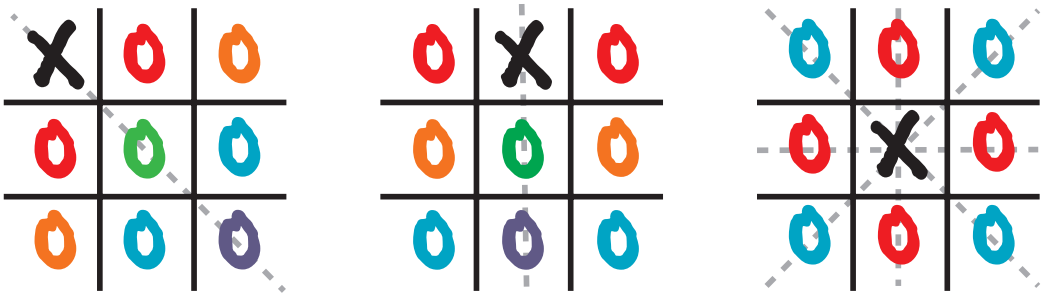


Figure 1

We can see from Figure 1 that there are 5 possible moves for Player B if Player A chooses a corner—**adjacent side**, **near corner**, **center**, **far side** and **far corner**. Similarly, if Player A chooses a side, then Player B has 5 choices—**adjacent corner**, **near side**, **center**, **far corner** and **far side**. If Player A chooses the center, then Player B has only 2 possible moves—**side** or **corner**. This means there are $5 + 5 + 2 = 12$ possible ways for the first 2 moves of the game to play out, not 72 ways! Using their knowledge of symmetry while playing the game will help students narrow down their unique choices.

Proof Writing and Logic

In game-theory terms, tic-tac-toe is a game that is played with perfect information, has no element of chance and is finite. This means, more or less, that a game of tic-tac-toe when played with perfect logic by both players will always result in a tie, or a cat's game. Based on the 12 possible game starts described in the previous section, Player A can guarantee a win or a tie if he or she makes the most logical choices. This is a great opportunity to have students practice proof writing and logical thinking! Have students start from one of the 12 scenarios and justify the next moves. This section will provide a couple of examples of proofs.

- ***Let's look at a game where Player A places an X in a corner and then Player B places an O in an adjacent side space as shown in Figure 2. Can Player A win? Prove your answer.***

  In order to have the advantage, Player A should choose a move that "forces" Player B to place a mark in a specific spot. Player A has 7 choices for the next move in this scenario. Of these, 3 will guarantee a win. Here is a proof for one of these 3 moves:

  If Player A moves to the other adjacent side **(1)**, then Player B must put an O in the near corner **(2)** to block. If Player A then places an X at the center **(3)**, that guarantees a win since Player A then has two directions in which to achieve three in a row **(4)**. No matter where Player B moves, Player A can make a winning move.

  See if your students can find and prove the other two winning moves. You can also have them prove that still other moves will not guarantee a win.

  Hint: Adjacent corner and center are the other 2 moves that guarantee a win for Player A.

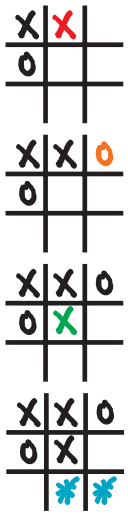


Figure 2

- ***Let's try one more proof. Suppose Player A starts at the center and then Player B marks a side. Can Player A have a guaranteed win?***

  In this game scenario, there are 4 options for Player A's next move—adjacent corner, near side, far corner and far side. Of these 4, 3 guarantee a win—adjacent corner, near side and far corner. A visual proof is given at the right, in Figure 3. You can also have students prove that the other move, far side, will result in a tie, or a cat's game.

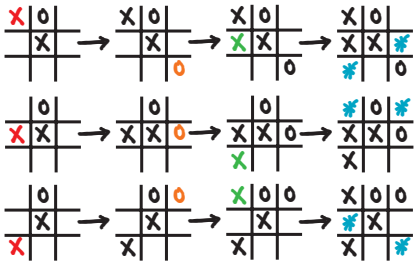


Figure 3

Differentiation, Scaling and Extensions

Change the Rules

Although we have presented three different ways to play tic-tac-toe, these are not all of the possible variations out there. If you want to try even more, here are a few to think about:

- Wild misere is a combination of the wild and misere rules. Players can choose any symbol each time, but the object is to NOT get three in a row.
- SOS is similar to wild tic-tac-toe, but players mark either an S or an O with the goal of being the first to form SOS. (*Note:* You can play with any three-letter palindrome of your choosing.)
- Play to n in a row on an $n \times n$ board to add a challenge or make the game longer. For example, play to four in a row on a 4×4 game board.

Extend Student Understanding

The math exploration here only skimmed the surface of what tic-tac-toe and the variations have to offer mathematically. Here are some other things to explore:

- Analyze symmetry and proofs with the variations. This activity focused on traditional rules, but there are opportunities to think logically with the other rules as well. As in the traditional game, is it impossible in a variation game to have a winner when neither player makes a mistake?
- Explore more of the combinatorics and game theory behind tic-tac-toe not addressed here.

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