THE MULTIPLICATION GAME

Everything You Need to Play

MATERIALS

The Multiplication Game Board

- 1 game board per pair of students. These instructions assume that students are playing one on one, but students can also play in two teams.
- Using a paper copy of the board is fine, but you may also want to make a laminated copy for multiple uses. A sheet protector is an easy substitute for lamination!

Writing Utensils

- 1 per student—colored pencils, regular pencils or dry erase markers (for laminated boards).
- If you use colored pencils (or dry erase markers), make sure paired students have two different colors. If you use regular pencils, students can mark squares with Xs and Os.

Paper Clips

- 2 per pair of students. Any small place markers, such as coins, could be used instead.

RULES

The Multiplication Game board consists of a 6-by-6 grid of products and a list of possible factors—the numbers 1 through 9. Two factors are multiplied to obtain one of the products. The two players compete to get four squares in a row—horizontally, vertically or diagonally.

- Decide who will be Player A and who will be Player B—use rock paper scissors, flip a coin, etc.
- To start, Player A puts a paper clip on one of the factors. Player A does not mark a square on the grid because only one factor has been marked; it takes two factors multiplied to make a product.
- Next, Player B puts the other paper clip on one of the factors (not excluding the same factor chosen by Player A) and then colors or marks the product of the two factors on the grid. (Figure 1)
- Player A moves one of the paper clips (not necessarily the same one Player A placed to start) to another factor and colors or marks the new product.
- Each player, in turn, moves one of the paper clips and marks the new product. If the product is already marked, the player does not make a mark for that turn.
- The winner is the first player to get four of his or her marked products in a row—horizontally, vertically or diagonally. (Figure 2)

Note: Similar to tic-tac-toe, there could be a "cat's game," in which no one can win. In that case, the game ends, and the two players should start a new game.

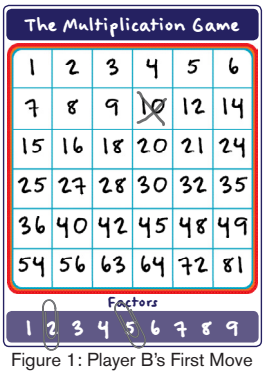

Figure 1: Player B's First Move

Figure 2: Completed Game

Differentiation, Scaling and Extensions

Change the Rules

Make things interesting by allowing students to vary the rules. Changing the rules will make students think through the effects of the new rules and adjust how they play the game. Rule changes can be as simple or complex as your club decides! Some examples for possible rule changes are:

- Make it five in a row to win the game instead of four.
- Allow any four connected squares to win (L-shape, square, etc.).
- Make it a game of three players instead of two.

Make a New Board

Once you discuss the composition of the board, students will have an understanding of why the board is the size it is and why the factors and products used were chosen. Have students use this knowledge to make their own boards. You can have different students or groups working on new boards that they can trade with each other. Some ideas for changing the board are:

- Use different factors, more factors, fewer factors, etc. This will change the size of the board and the numbers included.
- Repeat numbers so there are more choices of product squares for the same factor pair.
- Scramble the order of the products on the board so it's more challenging to establish a pattern or find the best area to get four in a row.
- Include products that aren't possible with the factors provided. Those products will create on the board a dead zone or blocked point that players have to identify.

THE MULTIPLICATION GAME

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors

1 2 3 4 5 6 7 8 9

THE MULTIPLICATION GAME

Mathematical Exploration

Game Board Composition

A good way to start a mathematical exploration of the Multiplication Game is by looking at the numbers on the game board and asking, "Why those numbers?" Although it is a 6-by-6 game board with 36 squares, the products included on the board aren't simply 1 through 36, inclusive. The products range from 1 to 81 with some numbers in between excluded. Guide your club through an exploration by asking them to think about and answer the following questions.

- ***What products are included on the game board, and what products are excluded from the game board? (Consider only the integers 1—81.)***

  Included: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 35, 36, 40, 42, 45, 48, 49, 54, 56, 63, 64, 72, 81

  Excluded: 11, 13, 17, 19, 22, 23, 26, 29, 31, 33, 34, 37, 38, 39, 41, 43, 44, 46, 47, 50, 51, 52, 53, 55, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80

- ***Which of the excluded products are prime numbers, and which are composite numbers?***

  Prime: 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79

  Composite: 22, 26, 33, 34, 38, 39, 44, 46, 50, 51, 52, 55, 57, 58, 60, 62, 65, 66, 68, 69, 70, 74, 75, 76, 77, 78, 80

- ***Why are these prime numbers excluded?***

  Students might notice that all the prime numbers between 1 and 81 are excluded except for 2, 3, 5 and 7. This is because a prime number's only factors are 1 and itself. So, although 1 is in the factor list, prime numbers greater than 7 are not.

- ***Why are these composite numbers excluded?***

  Although these composite numbers have more factors than the prime numbers, and many have factors in our factor list, none of the excluded composites can be expressed as a factor pair in which both factors are less than or equal to 9. You can have students go through the excluded composites and spot-check individual numbers to verify this.

- ***Are there any products with factor pairs consisting of two numbers from 1 through 9, inclusive, that are missing?***

  This is a way to introduce combinatorics. Students might make a chart similar to Figure 3 to illustrate all the combinations of factor pairs. All the possible products are included on the game board.

- ***How many ways can you choose a unique factor pair (the order of numbers in a factor pair does not matter)?***

  There are 45 unique factor pairs. Draw the diagonal line of symmetry, from the upper left corner to the bottom right corner, to demonstrate this. There are 9 factor pairs on the diagonal, 36 below the line, and "duplicates" of these 36 above the line. These are colored orange, purple and blue, respectively, in Figure 3.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Figure 3

Multiplication Game Strategies

Now that your students understand a little more about the game board's composition, you can have them analyze potential moves to develop a strategy for playing the game. Students will eventually notice that not all of the products are equally desirable or as easily obtainable.

- ***How many factor pairs does each product on the game board have?***

  In the previous exploration we discovered there are 45 unique factors pairs and 36 unique products. Therefore, some products have more than one factor pair associated with them. Ask students to write on the game board the factor pair(s) that use only the numbers 1 to 9, inclusive, below each product. The factor pairs are shown in Figure 4.

- ***What do you notice about the number of factor pairs?***

  Most of the products have only one factor pair, some of the products have two factor pairs, but none of the products have more than two factor pairs.

- ***Are some products easier to obtain than others in this game?***

  If we look closely at the factor pairs, we will notice that some products, 25 for example, can only be obtained using one number from the factor list, in this case double 5s. Other products can be obtained using two different numbers. For example, 10 has factors 2 and 5. The most numbers that any number on the board has as factors in the factor list are four unique numbers. For example, 18 is 2 × 9 and 3 × 6. If at least one paper clip is on one of those four factors when your turn comes, you can make the product 18. Figure 5 is color coded to show products with 1, 2, 3 and 4 unique factors.

- ***How does this observation affect your game strategy?***

  Products with more unique numbers as factors are going to be easier to obtain as a player and harder to block as an opponent. A player should aim to incorporate those products in a row of four. For example, it will be easier to obtain 8, 9, 10, 12 for four in a row than it will be to obtain 63, 64, 72, 81.

1 1 × 1	2 1 × 2	3 1 × 3	4 1 × 4, 2 × 2	5 1 × 5	6 1 × 6, 2 × 3
7 1 × 7	8 1 × 8, 2 × 4	9 1 × 9, 3 × 3	10 2 × 5	12 2 × 6, 3 × 4	14 2 × 7
15 3 × 5	16 2 × 8, 4 × 4	18 2 × 9, 3 × 6	20 4 × 5	21 3 × 7	24 3 × 8, 4 × 6
25 5 × 5	27 3 × 9	28 4 × 7	30 5 × 6	32 4 × 8	35 5 × 7
36 4 × 9, 6 × 6	40 5 × 8	42 6 × 7	45 5 × 9	48 6 × 8	49 7 × 7
54 6 × 9	56 7 × 8	63 7 × 9	64 8 × 8	72 8 × 9	81 9 × 9

Figure 4

1 1 × 1	2 1 × 2	3 1 × 3	4 1 × 4, 2 × 2	5 1 × 5	6 1 × 6, 2 × 3
7 1 × 7	8 1 × 8, 2 × 4	9 1 × 9, 3 × 3	10 2 × 5	12 2 × 6, 3 × 4	14 2 × 7
15 3 × 5	16 2 × 8, 4 × 4	18 2 × 9, 3 × 6	20 4 × 5	21 3 × 7	24 3 × 8, 4 × 6
25 5 × 5	27 3 × 9	28 4 × 7	30 5 × 6	32 4 × 8	35 5 × 7
36 4 × 9, 6 × 6	40 5 × 8	42 6 × 7	45 5 × 9	48 6 × 8	49 7 × 7
54 6 × 9	56 7 × 8	63 7 × 9	64 8 × 8	72 8 × 9	81 9 × 9

Figure 5