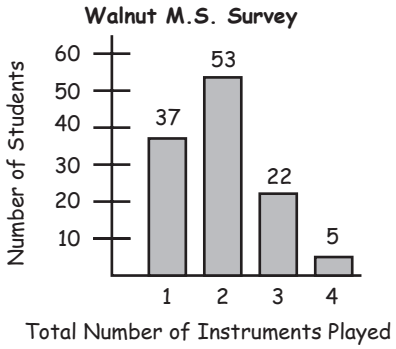
THE NATIONAL MATHCLUB

Statistics and Data

MATHCOUNTS has joined the Data Science 4 Everyone Commitment Campaign to build capacity for K-12 data science education. As part of this commitment, MATHCOUNTS will create and share additional resources to help students develop data science skills. The DS4E network, comprised of educators, content developers, policy experts and other advisers, works to raise awareness about data literacy, design curriculum, perform policy advocacy, and create resources for teachers and classrooms.

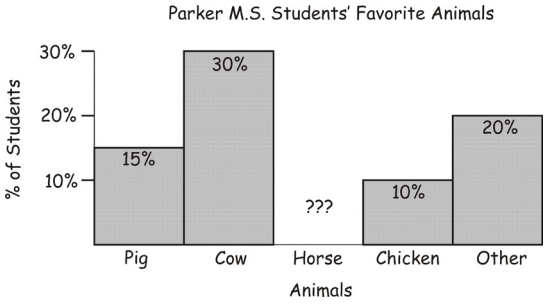
Data science is a critically important field, and there are many interesting careers in data science and analytics! *The great news for Mathletes is that the math you do in MATHCOUNTS—such as statistics, problem-solving, pattern recognition and probability—will prepare you to succeed in our data-driven world. Flex those math muscles with this Statistics & Data problem set!*

1. According to this bar graph, how many students surveyed at Walnut Middle School play two or more instruments?



2. Lee took a survey for the school newspaper and received the following data about the number of hours classmates studied per week: {1, 2, 4, 5, 8, 9, 10, 10}. What is the absolute difference between the median and mean of this set of data? Express your answer as a common fraction.

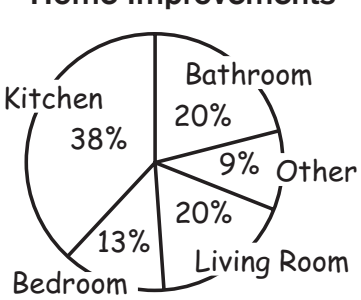
3. The students at Parker Middle School each submitted one vote for their favorite animal on Old MacDonald's Farm. The exact results are shown in this histogram. What percentage of the students voted for the horse?



4. The record of Mr. Guzzler's gas purchases during a four-week period is given in the table below. What was his average cost per gallon of gasoline purchased during the four-week period? Express your answer to the nearest cent.

	Week 1	Week 2	Week 3	Week 4
Price per Gallon	$2.99	$3.09	$3.29	$3.59
Number of Gallons Purchased	18	15	12	10

5. For a survey, 850 homeowners gave one answer to the question, "What would you most like to improve about your home?" The pie chart shows the percentage of the homeowners with particular responses, expressed to the nearest whole number. To the nearest 10 people, how many responded that they would improve their bedroom?



6. Twenty-five teachers were asked the length, in minutes, of their daily drive to school. Their responses are recorded in this stem-and-leaf plot. What percent of teachers take 25 or more minutes to drive to school?

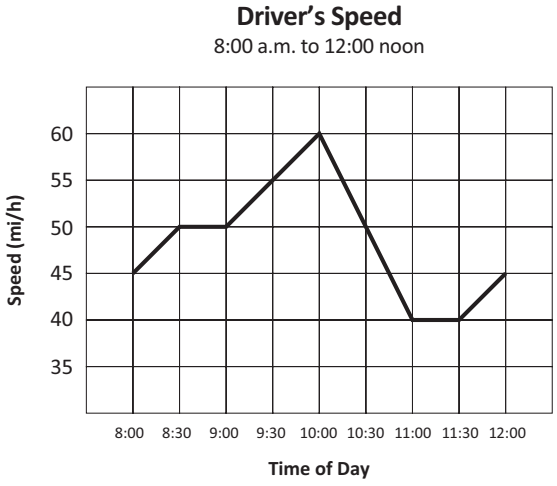
Driving Time to School (Minutes)

0	5 6 6 7 9
1	0 1 2 8 9 9
2	0 2 3 4 5 7
3	1 2 2 2 2 8
4	1 3

7. The four highest-scoring students in a class had scores of 97, 93, 90 and 80 on an exam. The average exam grade for the 12-member class was 75. What is the largest number of students who could have scored below 60?

8. In a recent survey of 300 students, 152 students had at least one dog, 120 students had at least one cat, and 46 students had at least one cat and at least one dog. How many of the surveyed students did not have either a cat or a dog?

9. The line graph shows one driver's speed, in miles per hour, from 8:00 a.m. to 12:00 noon. Based on the graph, what was the driver's average speed from 10:30 to 11:30? Express your answer as a decimal to the nearest tenth.



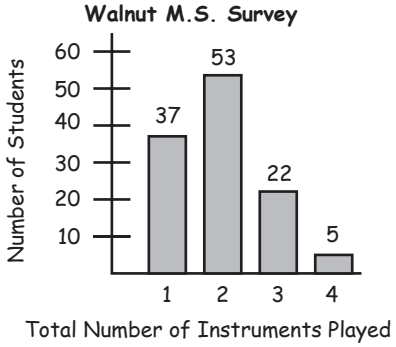
10. The mean, median and unique mode of six positive integers are 8, 7 and 3, respectively. What is the maximum possible value for the range of the six numbers?

Club Leader Solution Guide!

THE NATIONAL MATHCLUB

Statistics and Data

1. According to this bar graph, how many students surveyed at Walnut Middle School play two or more instruments?

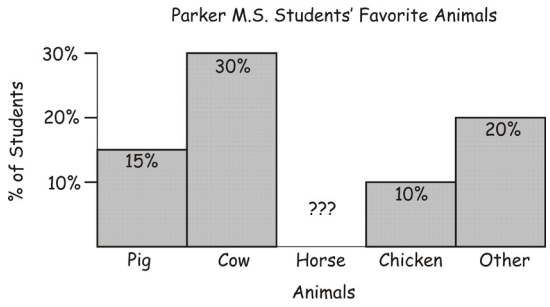


According to the bar graph, 53 + 22 + 5 = 80 students at Walnut Middle School play two or more instruments.

2. Lee took a survey for the school newspaper and received the following data about the number of hours classmates studied per week: {1, 2, 4, 5, 6, 8, 9, 10, 10}. What is the difference between the median and mean of this set of data? Express your answer as a common fraction.

The data is already listed in ascending order, which makes it easy to locate our middle value or median. The median is 6. To find our mean, we first sum the data and then divide by the total number of elements. The sum is 1 + 2 + 4 + 5 + 6 + 8 + 9 + 10 + 10 = 55. So, our mean is 55/9. The difference is 55/9 – 6 = 55/9 – 54/9 = 1/9.

3. The students at Parker Middle School each submitted one vote for their favorite animal on Old MacDonald's Farm. The exact results are shown in this histogram. What percentage of the studetns voted for the horse?



Each of the students voted for either the pig, cow, horse, chicken or another animal. Since every animal that is not a pig, cow, horse or chicken was put into the "other" category, 100% of the votes are recorded in the histogram. Between the pig, cow, chicken and "other" categories, the histogram shows 15% + 30% + 10% + 20% = 75%. The remaining 100% – 75% = 25% of the students must have voted for the horse.

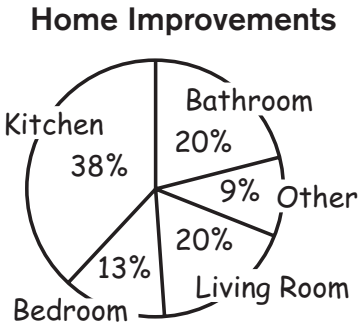
4. The record of Mr. Guzzler's gas purchases during a four-week period is given in the table below. What was his average cost per gallon of gasoline purchased during the four-week period? Express your answer to the nearest cent.

	Week 1	Week 2	Week 3	Week 4
Price per Gallon	$2.99	$3.09	$3.29	$3.59
Number of Gallons Purchased	18	15	12	10

Mr. Guzzler purchased a total of 18 + 15 + 12 + 10 = 55 gallons of gas. We will weight each price by the number of gallons purchased to compute a "weighted average" as follows: (18

× 2.99 + 15 × 3.09 + 12 × 3.29 + 10 × 3.59)/55 = (53.82 + 46.35 + 39.48 + 35.90)/55= 175.55/55 = $3.19.

5. For a survey, 850 homeowners gave one answer to the question, "What would you most like to improve about your home?" The pie chart shows the percentage of the homeowners with particular responses, expressed to the nearest whole number. To the nearest 10 people, how many responded that they would improve their bedroom?



We see that 13% of the homeowners said that they would improve their bedroom. We need to find 13% of the 850 homeowners surveyed. Converting the percent to its decimal and multiplying, we get 0.13 × 850 = 110 people, to the nearest 10 people.

6. Twenty-five teachers were asked the length, in minutes, of their daily drive to school. Their responses are recorded in this stem-and-leaf plot. What percent of teachers take 25 or more minutes to drive to school?

Driving Time to School (Minutes)

0	5 6 6 7 9
1	0 1 2 8 9 9
2	0 2 3 4 5 7
3	1 2 2 2 2 8
4	1 3

In the stem-and-leaf plot given, the tens digit of each number of minutes is written to the left of the vertical line and the ones digits for all numbers in that range are written to the right of the line. Fifteen of the teachers take less than 25 minutes to get to school, and 10 teachers take 25 minutes or more. Ten out of twenty-five (10/25) is equal to 40/100 or 40%.

7. The four highest-scoring students in a class had scores of 97, 93, 90 and 80 on an exam. The average exam grade for the 12-member class was 75. What is the largest number of students who could have scored below 60?

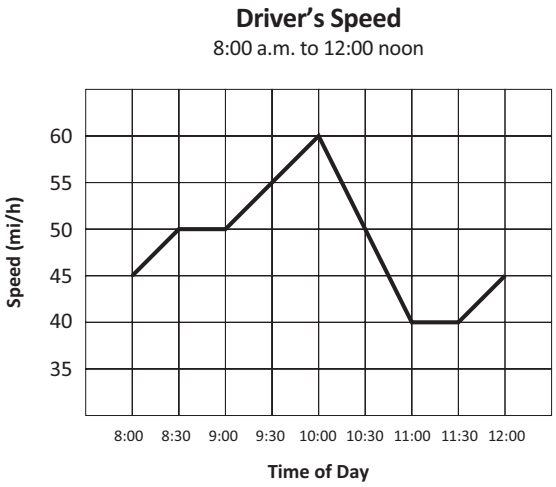
If the average of the 12-member class was 75, then the sum of their scores was 12 × 75 = 900 points. The four highest scores account for 97 + 93 + 90 + 80 = 360 of these points, so the remaining 8 students had a total score of 900 – 360 = 540 points. The average of these 8 students is 540 ÷ 8 = 67.5, so some of them must have scored above 60. The total number of points above a mean of 60 is 8 × 7.5 = 60 points. If we divide these 60 points among three students, then we would have three students with scores that are 20 points above 60, which is 80 points, and the other five students with scores of exactly 60. However, the five students should have scores less than 60, so this is not valid. We have learned that it is definitely possible for 4 students to score below 60.

8. In a recent survey of 300 students, 152 students had at least one dog, 120 students had at least one cat, and 46 students had at least one cat and at least one dog. How many of the surveyed students did not have either a cat or a dog?

If we add the number of students who have dogs to the number of students who have cats,

we have double counted the students who have both dogs and cats. Thus, there must be 152 + 120 − 46 = 226 students who have at least one of these pets. That leaves 300 − 226 = 74 students in the survey who did not have either a cat or a dog.

9. The line graph shows one driver's speed, in miles per hour, from 8:00 a.m. to 12:00 noon. Based on the graph, what was the driver's average speed from 10:30 to 11:30? Express your answer as a decimal to the nearest tenth.



According to the graph, the driver's speed was not constant throughout the hour from 10:30 to 11:30. During that time, the driver's speed decreased from 50 mi/h to 40 mi/h. The driver's speed decreased at a constant rate of change from 10:30 to 11:00. So, the average speed from 10:30 to 11:00 was (50 + 40)/2 = 45 mi/h. Then from 11:00 to 11:30 the driver's speed remained constant at 40 mi/h. From 10:30 to 11:30, the driver's average speed was (45 + 40)/2 = 42.5 mi/h.

10. The mean, median and unique mode of six positive integers are 8, 7 and 3, respectively. What is the maximum possible value for the range of the six numbers?

We will use the mean, median and unique mode to draw additional conclusions about these six numbers. Let *a*, *b*, *c*, *d*, *e* and *f* represent the six numbers, in ascending order. Since the sum of the six numbers is *a* + *b* + *c* + *d* + *e* + *f* = S, and the mean is 8, we can write S/6 = 8 → S = 48. We now know that the six numbers have a sum of 48. Since we have a group of six numbers, and there is no middle number, the median is the mean of the two middle numbers. In this case the median, which is 7, is the mean of *c* and *d*. We can write (*c* + *d*)/2 = 7 → *c* + *d* = 14. So, the sum of the two middle numbers is 14. Lastly, we know that 3 is the unique mode, which means at least two of our six numbers must be 3. Now our goal is to maximize the range of our six numbers, *f* − *a*. Since all six numbers are positive integers, the least possible value of one of these numbers is 1. We'll let *a* = 1. Since at least two of the six numbers must be 3, we'll let *b* = *c* = 3. Since *c* + *d* = 3 + *d* = 14, it follows that the value of *d* must be 11. The sum of the four numbers we've identified is 1 + 3 + 3 + 11 = 18. That means *e* + *f* = 48 − 18 = 30. Since we want the value of *f* to be as large as possible, we'll let the value of *e* be as small as possible, but not equal to *d* since 3 is the unique mode and only appears twice. Thus, *e* = 12 and *f* = 30 − 12 = 18. These six numbers, 1, 3, 3, 11, 12, 18, have a mean, median and unique mode of 8, 7 and 3, respectively. The maximum range for six positive numbers with these characteristics is 18 − 1 = 17.