MATHCOUNTS® Minis

October 2018 Activity Solutions

Warm-Up!

1. Because these are each arithmetic sequences, we know the difference between consecutive terms for each sequence remains constant. (In other words, the same amount is added to each term to get the next term.)

(a) The common difference is 11 – 5 = 6. The missing terms are 11 + 6 = 17, 17 + 6 = 23 and 23 + 6 = 29. The sequence is 5, 11, **17**, **23**, **29**.

(b) The common difference is 5 – 11 = –6. The missing terms are 5 – 6 = –1, –1 – 6 = –7 and –7 – 6 = –13. The sequence is **–13**, **–7**, **–1**, 5, 11.

(c) Going from 5 to 11, we must add the common difference to 5 a total of four times. is 6. Dividing the total difference of 11 – 5 = 6 into four equal parts, we get a common difference of 6/4 = 1.5. So, the missing terms are 5 + 1.5 = 6.5, 6.5 + 1.5 = 8 and 8 + 1.5 = 9.5. The sequence is 5, **6.5**, **8**, **9.5**, 11.

2. We can make 45 cents using any combination of quarters, dimes and nickels in the **8** ways shown.

25¢	1	1	1	0	0	0	0	0
10¢	2	1	0	4	3	2	1	0
5¢	0	2	4	1	3	5	7	9

3. If a four-digit number has exactly one 0 it must be in the units, tens or thousands place. In any of these three cases the remaining three digits can be 1, 2, 3, 4, 5,6,7,8 or 9. That means that there are (9 x 9 x 9 x 1) x 3 = 729 x 3 = **2187** such four-digit numbers.

4. Consider five dresser drawers labeled from top to bottom A through E. First, if exactly one drawer is open, the 5 possibilities are drawer A, B, C, D or E. Next, if exactly two drawers are open, the 6 possibilities are drawers A and C, A and D, A and E, B and D, B and E or C and E. Finally, there is only 1 possible way to have exactly three drawers open, drawers A, C and E. That’s a total of 5 + 6 + 1 = **12** ways in which one of more of the drawers can be opened to access the contents of each open drawer.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Let ABCD represent a finish order with Ashley first, Brett second, and so on. There are 4! = 24 possible finishing orders, not including ties. We can systematically consider these 24 orders as follows:

ABCD satisfies (1) but not (2). We count this order.
ABDC satisfies both (1) and (2). We eliminate this order.
ACBD satisfies (1) but not (2). We count this order.
ACDB satisifies both (1) and (2). We eliminate this order.
⋮

Continuing in this manner, we find that ABCD, ACBD, ADBC, ADCB, BADC, BDAC, BDCA, CADB, CDAB, CDBA, DABC and DACB are the **12** finishing orders for which exactly one of Eric's statements is correct.

6. There are two ways for a log to get from pond A to pond B: A → K → B or A → J → B.

Case 1: The probability that a log takes the route A → K → B is (1/3) × (1/2) = 1/6.

Case 2: The probability that a log takes the route A → J → B is (1/3) × (1/3) = 1/9.

Based on these two case, the probability of a log going from pond A to pond B is 1/6 + 1/9 = (3 + 2)/18 = **5/18**.

7. Let's start by determining which perfect squares can obtained by multiplying two different numbers from 1 to 16, inclusive. Since 15 × 16 = 240, the perfect squares in question will all be less than 240. The perfect squares less than 240 are 1, 4, 9, 16, 25, 36 ,49, 64, 81, 100, 121, 144, 169, 196 and 225. The perfect squares that can not be obtained by multiplying two different numbers from 1 to 16, inclusive are 1, 25, 49, 81, 100, 121, 169, 196 and 225. We can, however, obtain the following products, which are perfect squares:

4 = 1 × 4	9 = 1 × 9	36 = 3 × 12 = 4 × 9
16 = 1 × 16 = 2 × 8	64 = 4 × 16	144 = 9 × 16

To avoid drawing two numbers whose product is a perfect square, Jillian must not draw the numbers 1, 2, 3, 4, 8, 9, 12 and 16. So the first eight numbers Jillian can draw without a pair whose product is a perfect square are 5, 6, 7, 10, 11, 13, 14, 15. There are 2 ways for her to draw the ninth and tenth numbers without a pair whose product is a perfect square:

Case 1: If she draws 2 and 3, that leaves the numbers 1, 4, 8, 9, 12 and 16.

Case 2: If she draws 8 and 12, that leaves the numbers 1, 2, 3, 4, 9 and 16.

In either case, if Jillian next draws the 1, 4, 9 or 16, she will have drawn a total of 11 numbers without a pair whose product is a perfect square. Any number drawn after that will result in a pair of numbers whose product is a perfect square, so the maximum number of slips that Jillian can draw is **11** slips.

8. Let's start with the case of two blue marbles, since this is the fewest there could be, and add one blue marble for each new case.

Case 1: The two blue and three green marbles can be arranged the following ways: BBGGG, GBBGG, GGBBG, GGGBB. This case yeilds 4 arrangements.

Case 2: With three blue and two green marbles, all three blue must be together. We can arrange the marbles in the following ways: BBBGG, GBBBG, GGBBB.
This case yeilds 3 arrangements.

Case 3: With four blue marbles and one green, the blue marbles can either be in a group of four or two groups of two. The marbles can be arranged in the following ways: BBBBG, BBGBB or GBBBB. This case yields 3 arrangements.

Case 4: Finally, all the marbles could be blue: BBBBB. This case yields 1 arrangement.

The total of all the cases is 4 + 3 + 3 + 1 = **11** arrangements.