MATHCOUNTS® Minis January 2018 Activity Solutions

Warm-Up!

1. There are three different sized squares in the figure, 1 × 1, 2 × 2 and 3 × 3. There are 9 squares of size 1 × 1, 4 of size 2 × 2 and 1 of size 3 × 3, for a total of 9 + 4 + 1 = **14** squares.

2. We can make 45 cents using any combination of quarters, dimes and nickels in the **8** ways shown.

25¢	1	1	1	0	0	0	0	0
10¢	2	1	0	4	3	2	1	0
5¢	0	2	4	1	3	5	7	9

3. Consider five dresser drawers labeled from top to bottom A through E. First, if exactly one drawer is open, the 5 possibilities are drawer A, B, C, D or E. Next, if exactly two drawers are open, the 6 possibilities are drawers A and C, A and D, A and E, B and D, B and E or C and E. Finally, there is only 1 possible way to have exactly three drawers open, drawers A, C and E. That's a total of 5 + 6 + 1 = **12** ways in which one of more of the drawers can be opened to access the contents of each open drawer.

4. There are three triples that add up to 6. Let's examine these three cases:
Case 1: The triple 2, 2 and 2 can be ordered in 1 way.
Case 2: The triple 1, 1 and 4 can be ordered in 3!/2 = 3 ways.
Case 3: The triple 1, 2 and 3 can be ordered in 3! = 6 ways.
These three cases yield a total of 1 + 3 + 6 = **10** ordered triples.

The Problems are solved in the MATHCOUNTS® Mini video.

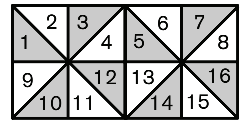
Follow-up Problems

5. Let ABCD represent a finish order with Ashley first, Brett second, and so on. There are 4! = 24 possible finishing orders, not including ties. We can systematically consider these 24 orders as follows:

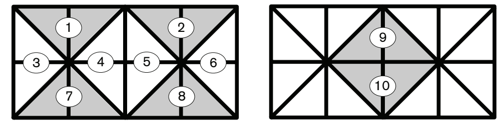
ABCD satisfies (1) but not (2). We count this order.
ABDC satisfies both (1) and (2). We eliminate this order.
ACBD satisfies (1) but not (2). We count this order.
ACDB satisifies both (1) and (2). We eliminate this order.
⋮

Continuing in this manner, we find that ABCD, ACBD, ADBC, ADCB, BADC, BDAC, BDCA, CADB, CDAB, CDBA, DABC and DACB are the **12** finishing orders for which exactly one of Eric's statements is correct.

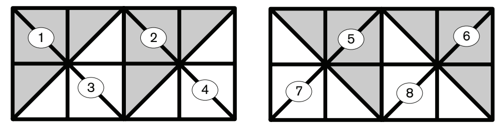
6. There are 16 triangles that contain no smaller triangles within them.



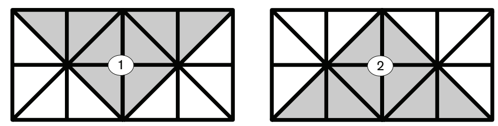
Now look for triangles that are made out of two smaller triangles. There are 10 of those.



There are a total of 8 triangles made out of four smaller ones.



The only other triangular shape is made out of eight smaller triangles. There are 2 of those.



That's a total of 16 + 10 + 8 + 2 = **36** triangles.

7. Let’s start with the case in which the sum includes five 1. Then we'll reduce the number of 1s while listing the sums for each case.

Case 1: In the case of five 1s, there is only 1 sum: 13 + 1 + 1 + 1 + 1 + 1.

Case 2: In the case of four 1s, the other two odd numbers must add up to 14. There are three possibilities: 11 + 3, 9 + 5 and 7 + 7. This case yields 3 sums.

Case 3: In the case of three 1s, the other three odd numbers must add up to 15. There are three possibilities: 9 + 3 + 3, 7 + 5 + 3 and 5 + 5 + 5. This case yields 3 sums.

Case 4: In the case of two 1s, the other four odd numbers must add up to 16. There are two possibilities: 7 + 3 + 3 + 3 and 5 + 5 + 3 + 3. This case yields 2 sums.

Case 5: In the case of one 1, the other five odd numbers must add up to 17. There is one possibility: 5 + 3 + 3 + 3 + 3. This case yields 1 sum.

Case 6: In the case of no 1s, there is one possibility with six odd numbers that add up to 18: 3 + 3 + 3 + 3 + 3 + 3. This case yields 1 sum.

These six cases yield a total of 1 + 3 + 3 + 2 + 1 + 1 = **11** sums.

8. Let’s start with the case of two blue marbles, since this is the fewest there could be, and add one blue marble for each new case.

Case 1: The two blue and three green marbles can be arranged the following ways: BBGGG, GBBGG, GGBBG, GGGBB. This case yeilds 4 arrangements.

Case 2: With three blue and two green marbles, all three blue must be together. We can arrange the marbles in the following ways: BBBGG, GBBBG, GGBBB. This case yeilds 3 arrangements.

Case 3: With four blue marbles and one green, the blue marbles can either be in a group of four or two groups of two. The marbles can be arranged in the following ways: BBBBG, BBGBB or GBBBB. This case yields 3 arrangements.

Case 4: Finally, all the marbles could be blue: BBBBB. This case yields 1 arrangement.

The total of all the cases is 4 + 3 + 3 + 1 = **11** arrangements.